Modeling and Stochastic Analysis of Distributed Control Problems to Meet Hard Real-Time SLA Conditions

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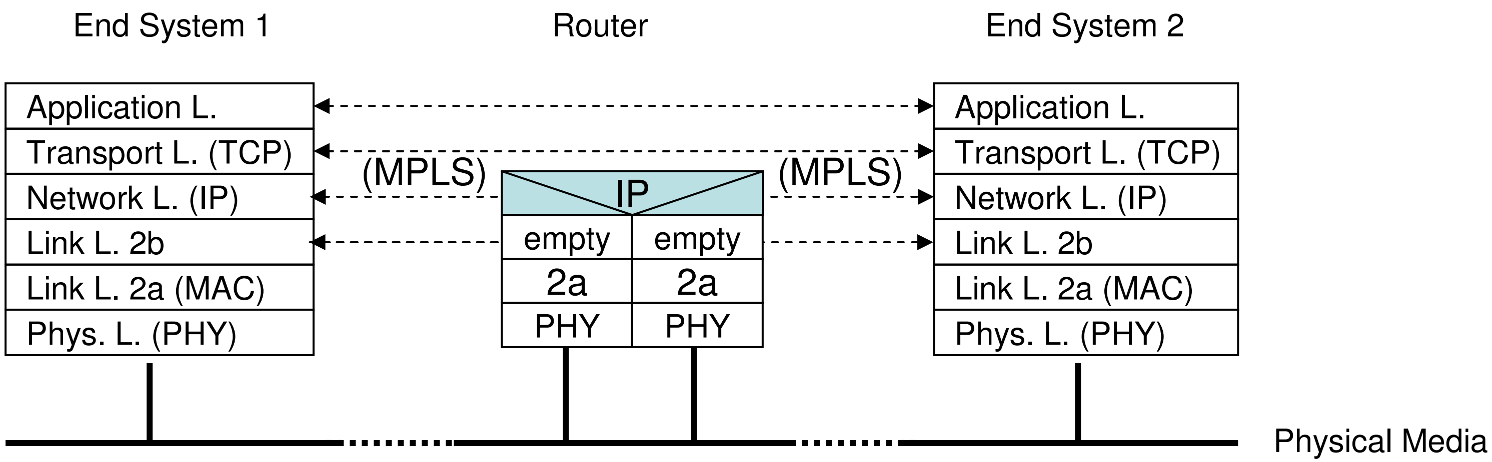
Outline

- Multi-Layer Protocol Architectures and Applications
- Link-Layer Protocol Modeling
- Performance Evaluation by Task Graph Analysis and Queuing Model Aggregation
- Application to Networked Control Systems (NCS)
- Conclusions and Outlook

1 Multi-Layer Protocol Architectures and Applications

Internet Protocol Architecture



Applications

Voice/Video/Data Communication End-to-End

Web Access Services

Distributed Cloud Services

Distributed Control Applications: Cyber-Physical Systems (CPS) with Applications in Smart Grid, Integrated Production Systems, Traffic Control, …

Example: Real-Time Performance of Networked Control Systems (NCS)

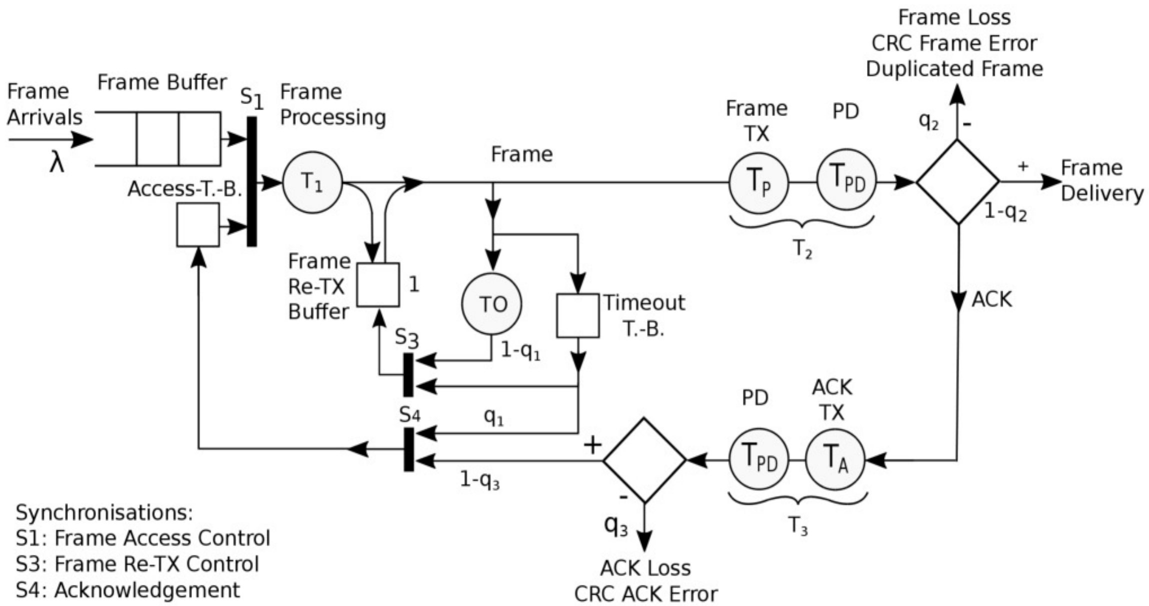
1 Multi-Layer Protocol Architectures and Applications

- Requirements in Cyber-Physical Systems
  - Reliability, Dependability, Security, Management, …
  - Performance: expecially **Quality of Service** (QoS) under **Service Level Agreements** (SLA)
- Methodology applied in this Study
  - Layer-Specific Protocol Models ⇨ Hybrid Task Graph Representation
  - Stochastic Task Graph Reduction ⇨ Queuing Model Representation
  - Queuing Model Analysis ⇨ Aggregated Layer Performance
  - Multi-Layer Protocol Models ⇨ Successive Aggregation Methodology
- Application Example
  - Layer 2b Protocol Models for „Stop- and Wait“ (SW) and „Selective Repeat“ (SR) Protocols
  - Application to Networked Control Systems (NCS) as Application Layer

2 Link-Layer Protocol Modeling

2.1 Send-and-Wait (SW) Protocol with ACK/TO Control

Model



Parameters

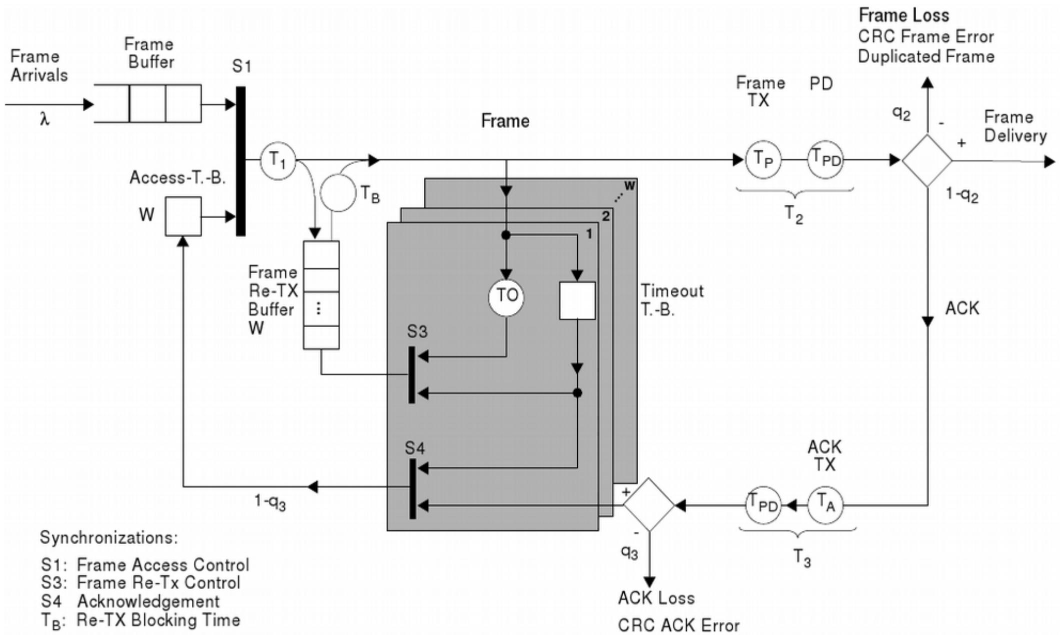
Frame Arrivals	G(M)
Frame Buffer Capacity	$0 \le s \le \infty$
Frame Processing	G(D), T_1
Frame Transmission	G(D), Tp
Propagation Delay	D, T_{PD}
Ack. Frame Transm.	G(D), T_A
Timeout time	D, T0
T0 Recovery Prob.	q_1
CRC Frame Error Prob.	q_2
CRC ACK Error Prob.	q_3

- Synchronisation Conditions indicated by Petri-Net Symbols and Token Buffers
- Disadvantage of the SW-Protocol: **Throughput Limitation**
- Aim of the Analysis: **Virtual Frame Transmission Time T_X**

2 Link-Layer Protocol Modeling

2.2 Selective –Repeat (SR) Protocol with ACK/TO Control

Model



Parameters

- System Parameters:
  as for the SW-Protocol
- Additional Parameter:
  Window Size w of the SR-Protocol

  $0 \leq w \leq 2^{m-1}$

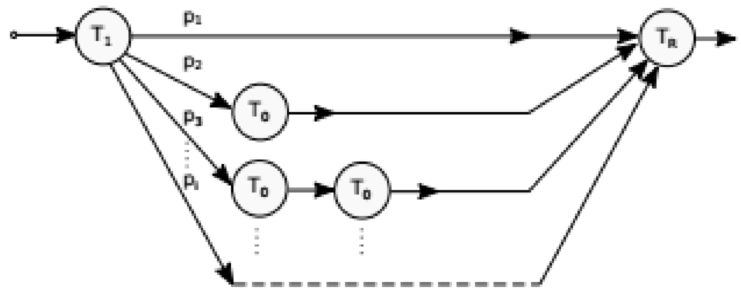
  for cyclic Frame Numbers out of the Internal $[0,2^m-1]$,
  $m = 1, 2, \ldots$

- Synchronization for each admitted Frame identical as in Case of the SW Protocol
- Virtual Transmission Times T_X are **iid-Variables** for each admitted Frame
- Advantage of the SR-Protocol: **Throughput Extension**

3 Performance Evaluation by Task Graph Analysis and Queueing Model Aggregation

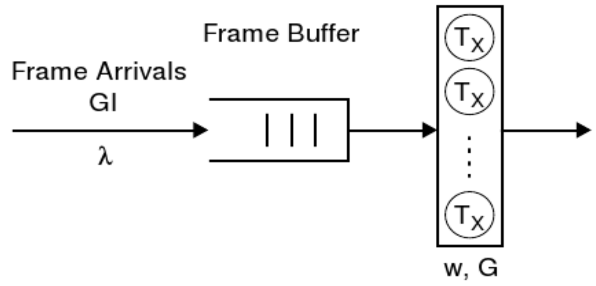
3.1 Task Graph for SW Protocol with ACK/TO

- Task Graph for T_X composition:



- A successful Frame Transmission Cycle happens only when the Frame **and** the ACK are successfully transmitted **and** if Cycle Time $T_0 = T_2 + T_3 < T0$ where T0-Recovery Probability $q_1 = P\{T_2 + T_3 > T0\}$
- The Frame fails successful transmission with Failure Probability $q_F = 1 - (1 - q_1)(1 - q_2)(1 - q_3)$
- Phase $T_0 = T_p + T_A + 2T_{PD}$
- Phase $T_R = \{T_0 | T_0 < T0\}$

3.1 Task Graph for SW Protocol with ACK/TO



- Virtual Frame Transmission Time T_X
  → Service Time of a Queuing System
- Queuing System Type

GI/G/1	for SW-Protocol
GI/G/w	for SR-Protocol

3.3 Analysis of Virtual Frame Transmission Time T_X (Constant Frame Sizes)

- Random Number N of Frame Transmissions:

PD: $p_n = P\{N = n\} = q_F^{\,n-1}(1 - q_F), \quad n = 1,2,\ldots$

RV: $T_X(n) = \quad T_1 + (n-1)T0 + \{T_2 + T_3 \mid T_2 + T_3 \le T0\}.$

PDF: $f_X(t) = \quad (1 - q_F)\sum_{i=1}^{\infty} q_F^{\,i-1}\delta(t - [t_1 + it_0]).$

Average: $E[T_X] = \quad t_1 + t_0 + \frac{T0}{1 - q_F} q_F.$

Moments: $E[T_x^{\,n}] = (1 - q_F)\sum_{i=1}^{\infty} q_F^{\,i-1}[t_1 + t_0 + (i-1)T0].$

max. Rate: $\lambda_{max} = \quad 1/E[T_X] = 1/t_X.$

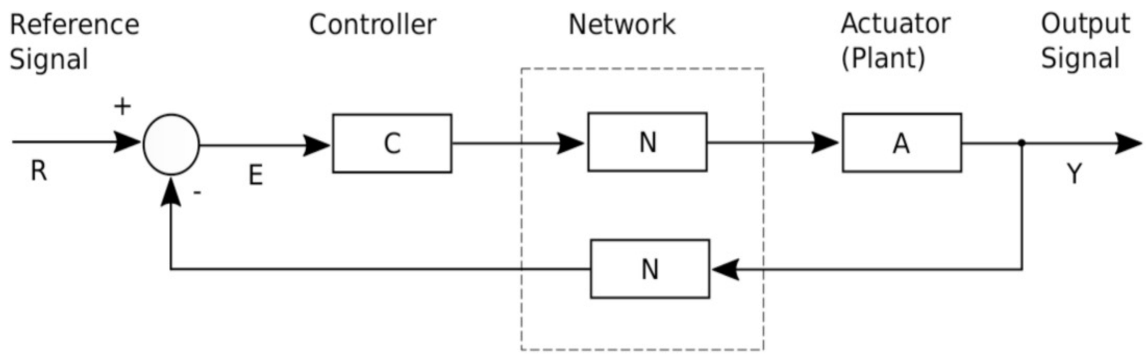
3.4 Queuing System Analyses

• **For Queuing Model GI/G/n:**	no exact Solution available	
• **Exact Solutions are known for:**	GI/M/c GI/M/c M/D/c	
• **Useful Approximations:**	GI/G/1 M/D/c	2-Moment Approximations Output Process
• **Queuing System Tables:**	$M/E_k/n$, M/D/c, D/M/c, $E_k/E_m/c$	F.S. Hiller, M.I. Reimann Stanford University, 1981, USA
	30 Queuing Systems	P. J. Kühn University of Stuttgart, 1976, D
	GI/G/c Approx.	L.P.Selen, H.C.Tijms, M.H. van Hoorn Free University Amsterdam, 1985, NL
	GI/G/c	W. Whitt Bell Laboratories, 1993, USA

4 Application to Networked Control Systems (NCS)

4.1 Distributed NCS with Embedded Networks (Aggregated Link Layer Protocol Models)

- Control Loop with Embedded Network



- Network N Representation as a Bi-Directional Layer 2b Subsystem Model with SW-Protocol between Controller C and Input of Actuator A and between the Output of Actuator A and Decision Operator of the Controller C, respectively
- Representation of the two Embedded N-Models by Queuing Systems of Type GI/G/1, where

GI = { D Deterministic Sampling Input Process of Plant System State (Time-driven Control)
M State-Dependent Level-Crossing of Plant System State (Event-driven Control)

G = Flow Time (i.e., Waiting + Service Time) of the Protocol Queueing Model GI/G/1

4 Application to Networked Control Systems (NCS)

4.2 Real-Time Performance of the NCS for Event-driven Control

• **Aggregated Queueing Model M/G/1**		**Dependent on**
W	Probability of Delay	ρ
$E[T_w]$	Mean Waiting Time	ρ, $E[T_X]$, c_X^2
$E[T_w^2]$	Second Moment of Waiting Time	ρ, $E[T_X]$, $E[T_X^2]$, $E[T_X^3]$
$E[T_F]$	Mean Flow (Sojourn) Time	ρ, $E[T_X]$, $E[T_W]$
c_F	Coefficient of Variation of Flow Time	ρ, $E[T_P]$, $E[T_F^2]$
c_D	Coefficient of Variation of the Delayed Frames	ρ, $E[T_X]$, $E[T_X^2]$, $E[T_X^3]$
$W^c(t)/W$	Compl. DF of Waiting Times of Delayed Frames Approximated by Weibull-Distribution Function	
t_{Th}	Threshold Value for Real-Time Delays	
p	Percentile of Delayed Frames $p = W^c(t_{Th})/W$	

4 Application to Networked Control Systems (NCS)

4.2 Real-Time Performance of the NCS for Event-driven Control

- **Network and Protocol Parameters**

Constant Frame Processing Time (T_1)	t_1	=	0.1 ms
Constant Frame and ACK Transmission Times (T_P, T_A)			
Constant Propagation Delay Time (T_{PD})			
Cycle Time $T_0 = T_P + T_A + 2T_{PD}$	t_0	=	1 ms
Frame Failure Probability	q_F	=	0.1
Frame Arrival Rates λ	λ	=	0.1, …, 0.75 Frames/ms
Percentiles	p	=	0.05, 0.02, 0.01
Protocol SW with ACK/T0 Control	T0	=	1.5 ms

- **Control System Parameters**

Controller Type: PID $C(s) = P + I/s + Ds$	P	=	0.21, I = 0.344, D = 0.03
Plant System Function A(s)	A(s)	=	$1000/(s^2 + s)$

4 Application to Networked Control Systems (NCS)

4.3 Numerical Results and Verification by Simulations

- **Verification of Results for SW Protocol with ACK/TO Control**

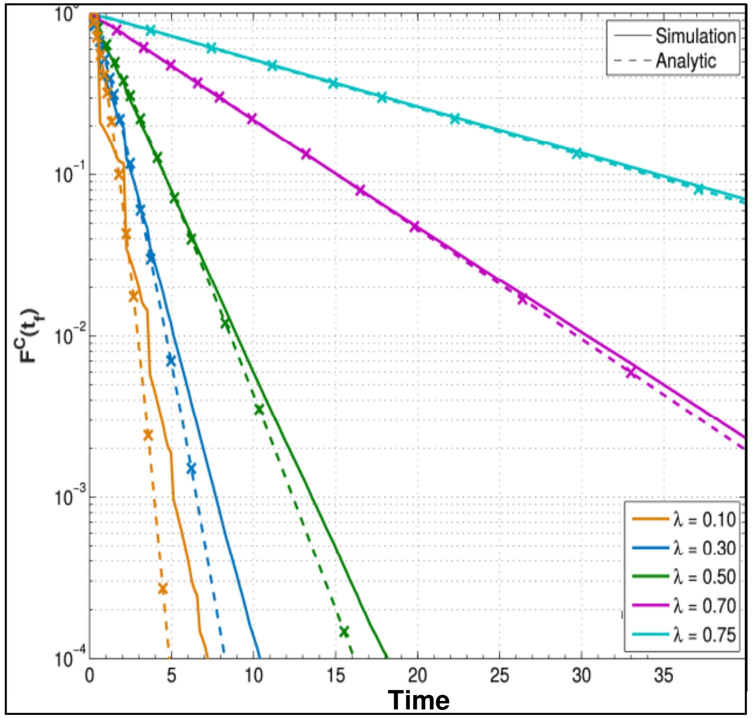
Table 1. Results of the ACK/TO SW Protocol Analysis Together with Computer Simulation Results

SW Protocol with Ack/Timeout Control						
λ /n		0.10	0.30	0.50	0.70	0.75
ρ		0.127	0.380	0.633	0.887	0.950
$E[T_X]$	analytical	1.267	1.267	1.267	1.267	1.267
	simulation	*1.266*	*1.267*	*1.267*	*1.267*	*1.267*
$E[T_W]$	analytical	0.108	0.455	1.283	5.812	14.117
	simulation	*0.108*	*0.457*	*1.290*	*5.856*	*15.093*
t_D	analytical	0.851	1.198	2.026	6.556	14.860
	simulation	*0.853*	*1.210*	*2.034*	*6.660*	*14.346*
c_D	analytical	0.867	0.907	0.946	0.984	0.993
	simulation	*0.872*	*0.911*	*0.949*	*0.988*	*0.995*
$E[T_F]$	analytical	0.894	1.241	2.069	6.599	14.903
	simulation	*0.894*	*1.244*	*2.076*	*6.642*	*15.133*
c_F	analytical	0.731	0.831	0.912	0.976	0.990
	simulation	*0.733*	*0.833*	*0.914*	*0.980*	*0.992*

4 Application to Networked Control Systems (NCS)

4.3 Numerical Results and Verification by Simulations

- **Compl. DF of Delayed Frames**



Complementary DF of the Flow Time T F for Five Different Load Levels together with Simulation Results

- **Delay Thresholds for**
  - **- 5 different Load Cases**
  - **- 3 different Percentile Values**

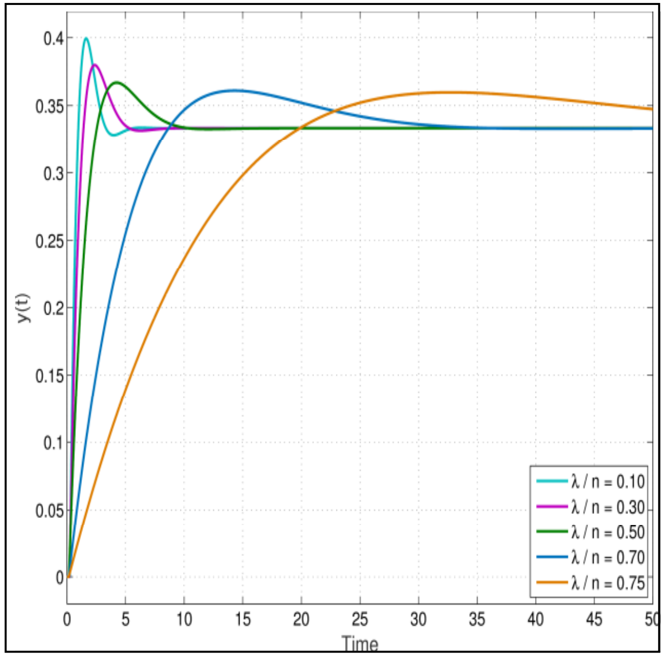
Table 1. Delay Thresholds t_{Th} for Three Different Percentiles p

λ 1/ms	0.10	0.30	0.50	0.70	0.75
t_{Th} ms, p = 0.05	2.1	3.9	5.9	19.7	42.2
t_{Th} ms, p = 0.02	2.4	4.2	8.9	24.7	57.8
t_{Th} ms, p = 0.01	2.8	5.0	9.2	29.8	67.8

4 Application to Networked Control Systems (NCS)

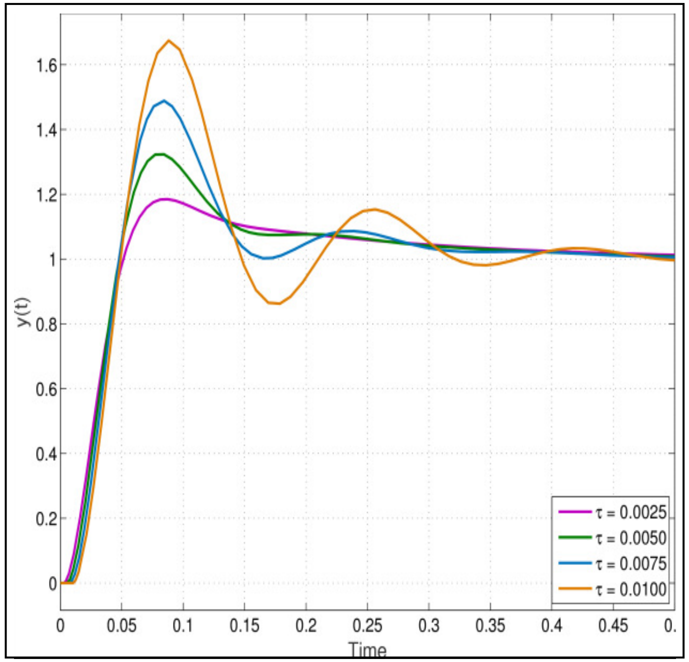
4.3 Numerical Results and Verification by Simulations

- **Unit-Step Function Responses y(t) for different Load Levels**



Unit-Step Function Responses of the NCS with SW LLC Protocol for Five Load Levels λ for n = 1 Server

- **Unit-Step Function Responses for Plant System Control for 4 Different Constant Network Delays τ**



Unit-Step Responses for Four Constant Network Delays Indicated in the legend

5. Conclusions and Outlook

Conclusions

- Novel Approach for a Comprehensive Link Layer Protocol Model Using Logical PN Symbols within a Task Graph Representation
- Exact Mathematical Performance Analysis
  Based on a Method of Stochastic Task-Graph Reduction Method
- Aggregation of a Complete Protocol Layer by a Stochastic Random Variable as Basis for a stepwise Aggregation of Multi-Layer Protocol Architectures
- Method has been demonstrated for Networked Control Systems (NCS)
  across common Layer 2b-Protocols SW and SR with ACK/TO Control to analyse in particular their Real-Time Performance

Outlook

- Extension of Method to NCSs with Shared Common Network Infrastructures as
  - shared MAC-Layer Protocols
  - shared Internet TCP Layer Protocol