Average Channel Gain Prediction Based on Spatial-Temporal Correlation

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Outline

1. Introduction
2. System Model
3. Modeling The Average Channel Gain
4. Model-Based Inference and Prediction
5. Empirical Evaluation
6. Summary

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Introduction

- Objective
  - Robust prediction of average channel gain, for both short-term and long-term
- State of the art
  - Prediction with time series forecasting methods to capture the temporal correlation
    - Disadvantage: aiming at short-term prediction
  - Prediction with Gaussian process (GP) or support vector machine (SVM) aided by location information to capture the spatial correlation
    - Disadvantage: aiming at mid- and long-term prediction, sensitive to estimation error about locations
- Motivation
  - Slow fading is spatially correlated for static users.
  - Temporal correlation is introduced when moving users lead to a time-variant pathloss.
  - Fast fading is temporally correlated once relative motion is introduced between any of the TXs or RXs.
  - Predictive model: Bayesian regression with spatial and temporal (BRST) correlation

Predictive model:

autoregressive process (temporal) + functional linear regression (spatial)

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System Model

- Instantaneous channel gain report
  - $h_i = l_i \cdot r_i$, composed of slow fading component l_i and fast fading component $r_i, i \in N$
- Average channel gain
  - H_n, L_n, R_n are the average of a number of N_0 measurements
  - $\ln H_n \approx \ln L_n + \ln R_n$, assuming independence between slow and fast fading
- Average fast fading
  - Sequence $\{\ln R_1, \ln R_2, \cdots\}$ of r.v. $\tilde{R}$ approximated follows the following distribution

Proposition (Approximated distribution of average fast fading)

Given that $r_1, r_2, \ldots$ are i.i.d. exponentially distributed with rate parameter λ, and that $r_i \in \mathbb{R}_{++}, \forall i$, then for N_0 approaches infinity, $\sqrt{N_0}\left(\tilde{R} - \ln\frac{1}{\lambda}\right) \xrightarrow{\mathcal{D}} \mathcal{N}(0,1)$, where $\xrightarrow{\mathcal{D}}$ denotes convergence in distribution. This leads to

$$\tilde{R} \approx \mathcal{N}(\ln\frac{1}{\lambda}, \frac{1}{N_0}), \qquad (1)$$

where here $\approx$ means "approximately distributed as" with slight abuse of notation.

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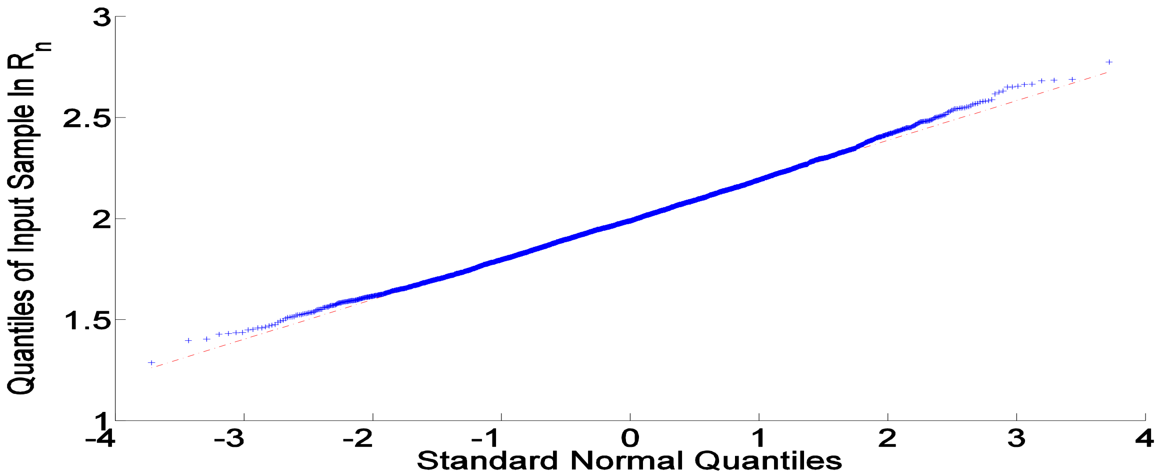

Fig. The q-q plot

Outline

Modeling The Average Channel Gain

- Training samples
  - Raw samples
    - channel gain measurements $\{z_1, z_2, \ldots\}$, where $z_n := \ln H_n$
    - Location estimation $\{s_1, s_2, \ldots\}$
  - Inputs (autoregressive channel gain & location estimation)

$$x_n := (z_{n-p+1}, \ldots, z_n, s_n^T)^T \in \mathbb{R}^{p+2}, n = 1, 2, \ldots$$

  - Outputs (d-step ahead prediction on average channel gain)

$$y_n := (z_{n+1}, \ldots, z_{n+d})^T \in \mathbb{R}^d, n = 1, 2, \ldots$$

- Collected sample set $\mathcal{S} := \{(x_n, y_n)\}_{n=1}^m$

- Adaptive selection of training samples
  - A union of the subsets $\mathcal{S}_{n,k} := \mathcal{S}_{n,k}^T \bigcup \mathcal{S}_{n,k}^S$
    - $\mathcal{S}_{n,k}^T$: from the time series of user k during the last N_{th} time period
    - $\mathcal{S}_{n,k}^S$: from the historical users with maximum difference of spatial distance D_{th}

Modeling The Average Channel Gain

- Functional linear regression model

$$y_n = f(x_n) = Ah(x_n) + W\phi(x_n) + \epsilon_n, \ \ n = 1, \ldots, m$$

  - Term $Ah(x_n)$: autoregressive model of order p to model the nonzero mean of the slow fading

$$h(x) := (e_1^T x, \ldots, e_p^T x)^T$$

  - Term $W\phi(x_n)$: a linear combination of orthonormal basis functions to specify a zero-mean slow fading distribution, with coefficient matrix W and basis function ϕ
  - Term $\epsilon_n := (\epsilon_{n,1}, \ldots, \epsilon_{n,d})^T$: a vector of i.i.d. Gaussian r.v.s with $\epsilon_{n,j} \sim \mathcal{N}(\ln(1/\lambda_j), 1/N_0)$ (using Proposition 1)

- Reformulation

$$\begin{aligned} y_n &:= [A\ \mu] \begin{bmatrix} h(x_n) \\ 1 \end{bmatrix} + W\phi(x_n) + (\epsilon_n - \mu) \\ &= \underbrace{\tilde{A}\tilde{h}(x_n)}_{\text{non-zero mean autoregressive process}} + \underbrace{W\phi(x_n) + \tilde{\epsilon}_n}_{\text{zero-mean GP}} \\ &= \tilde{A}\tilde{h}(x_n) + g(x_n). \end{aligned}$$

GP incorporated with explicit functions

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Model-Based Inference and Prediction

- Incorporating explicit functions in Gaussian process
  - Suppose conjugate prior $W \sim \mathcal{MN}_{d,q}(\mathbf{0}_{d\times q}, U, V)$
  - Replacing inner product in high-dimensional feature space with kernels

$$g(x) \sim \mathcal{GP}\left(\mathbf{0}, k(x,x')\cdot U + \frac{1}{N_0}\delta_{xx'}I_d\right)$$

  The joint distribution of $\{g(x_n)\}_{n=1}^{m}$ is written as

$$g_{md}|X \sim \mathcal{N}(\mathbf{0}, K\otimes U + \tfrac{1}{N_0}I_{dm}) \text{ where } (K)_{i,j} := k(x_i, x_j).$$

  - Marginal likelihood of f_{md} over g_{md}

$$f_{md}|X,\tilde{A} \sim \mathcal{N}(\mathrm{Vec}(\tilde{A}\tilde{H}), K\otimes U + \frac{1}{N_0}I_{dm})$$

- Marginal likelihood and inference of parameters
  - Suppose conjugate prior $\tilde{A} \sim \mathcal{MN}_{d,l+1}(M, U, Q)$
  - Log-marginal likelihood of y_{md} with prior on W and $\tilde{A}$ is derived by

$$p(f_{md}|X) = \int p(f_{md}|X,\tilde{A})p(\tilde{A}|X)d\tilde{A}$$

  - Two approaches to estimate the parameters $\theta := \{M, U, Q, \beta\}$:
    1) maximum marginal likelihood (ML), and 2) maximum a posteriori (MAP)

Model-Based Inference and Prediction

- Predictive Distribution
  - The conditional mean and variance of f_* given set of training samples $\mathcal{S}$ are written as

$$f_{*|\mathcal{S}} := f_*|y_{md}, X \;\sim \mathcal{N}\left(m_{*|\mathcal{S}}, \Sigma_{*|\mathcal{S}}\right)$$

$$\boxed{m_{*|\mathcal{S}} = m_* + \Sigma_* \Sigma^{-1}(y_{md} - m_{md})}$$

$$\Sigma_{*|\mathcal{S}} = \Sigma_{*,*} - \Sigma_* \Sigma^{-1} \Sigma_*^T.$$

Predictor

  - Where $m_{md} := \mathrm{Vec}(M\tilde{H})$ and $m_* := M\tilde{h}(x_*) \in \mathbb{R}^d$
  - The covariance matrices are

$$\Sigma := (\tilde{H}^T Q \tilde{H} + K) \otimes U + \tfrac{1}{n_0} I_{md}$$

$$\Sigma_* = (c(x_*, x_1), \ldots, c(x_*, x_m)) \otimes U \in \mathbb{R}^{d \times md}$$

$$\Sigma_{*,*} := c(x_*, x_*) \cdot U + (1/n_0) I_d \in \mathbb{R}^{d \times d}$$

  - Where the scalar function $c(x, x')$ is defined by $c(x, x') := \tilde{h}(x)^T Q \tilde{h}(x') + k(x, x')$.

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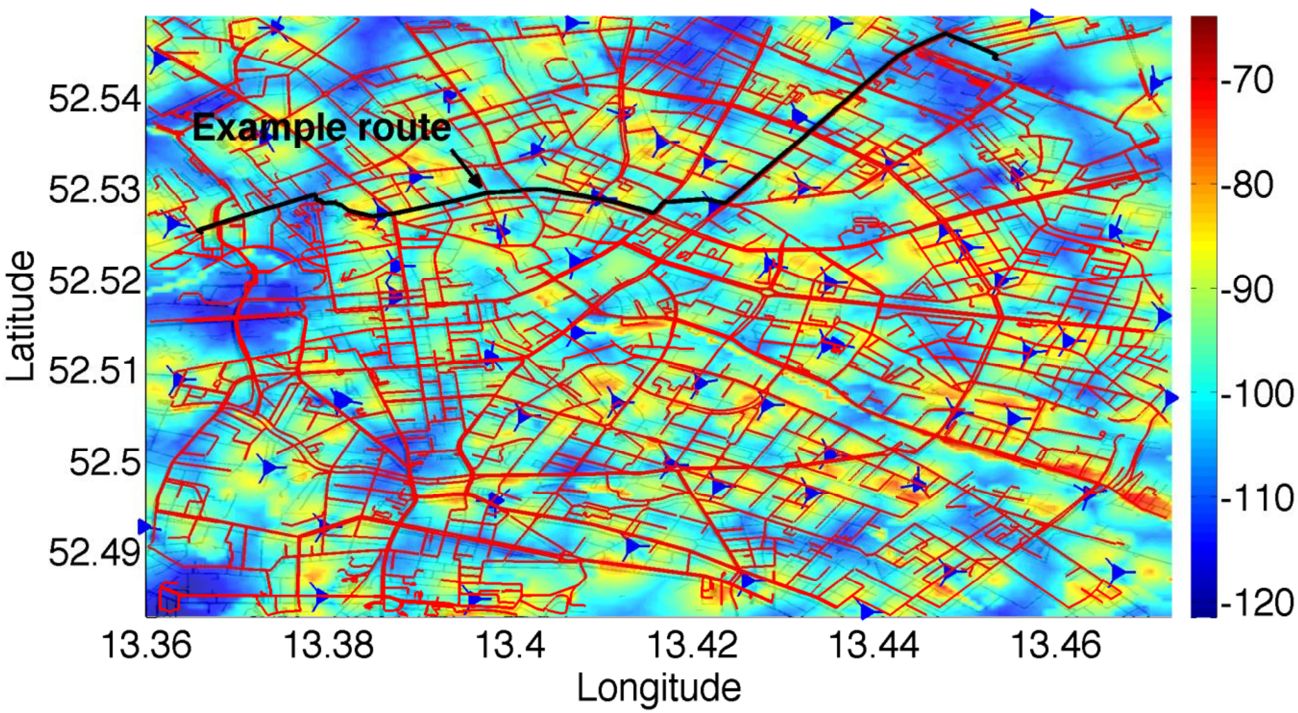
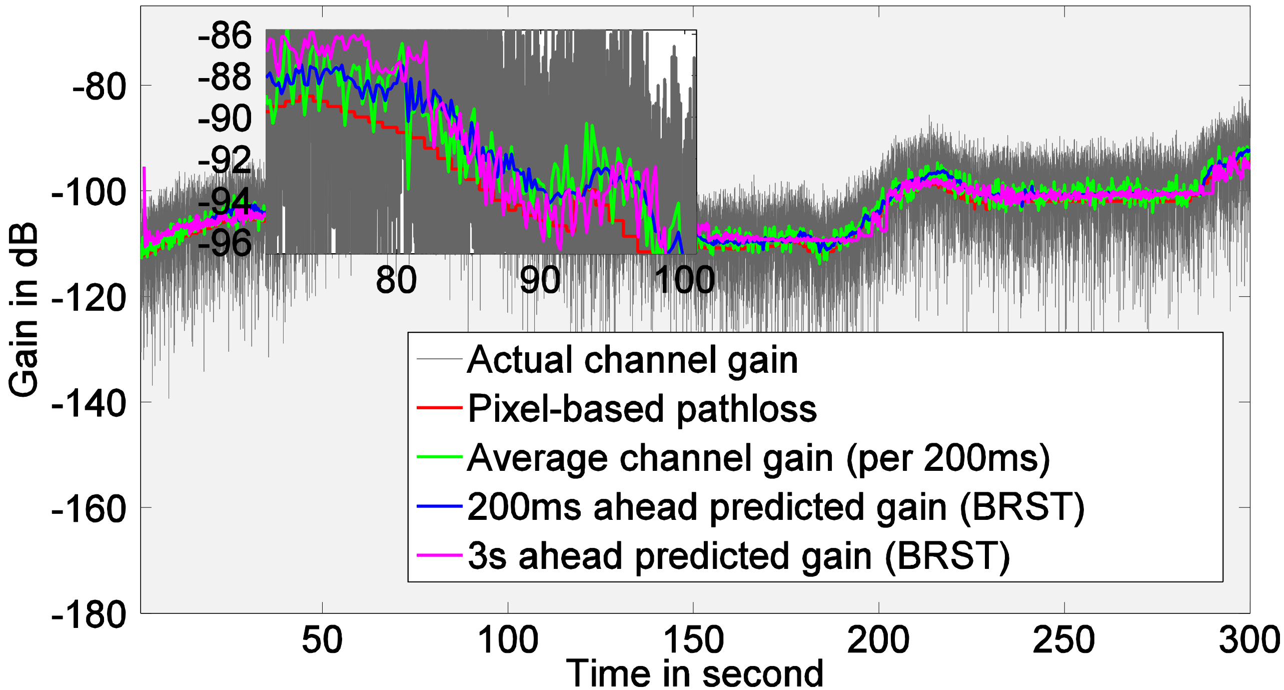
Empirical Evaluation

- Simulation scenario
  - Street map of center of Berlin, Germany (from *OpenStreetMap*)
  - Pixel-based pathloss data is obtained from *MOMENTUM* and correlated Rayleigh fading
  - Mobility of 1000 users at a maximum of 30 km/h using *SUMO*



Empirical Evaluation

- Performance of prediction wit BRST
  - Real-time channel gain is difficult to predict due to the large variations
  - BRST is able to predict short-term and long-term average channel gain



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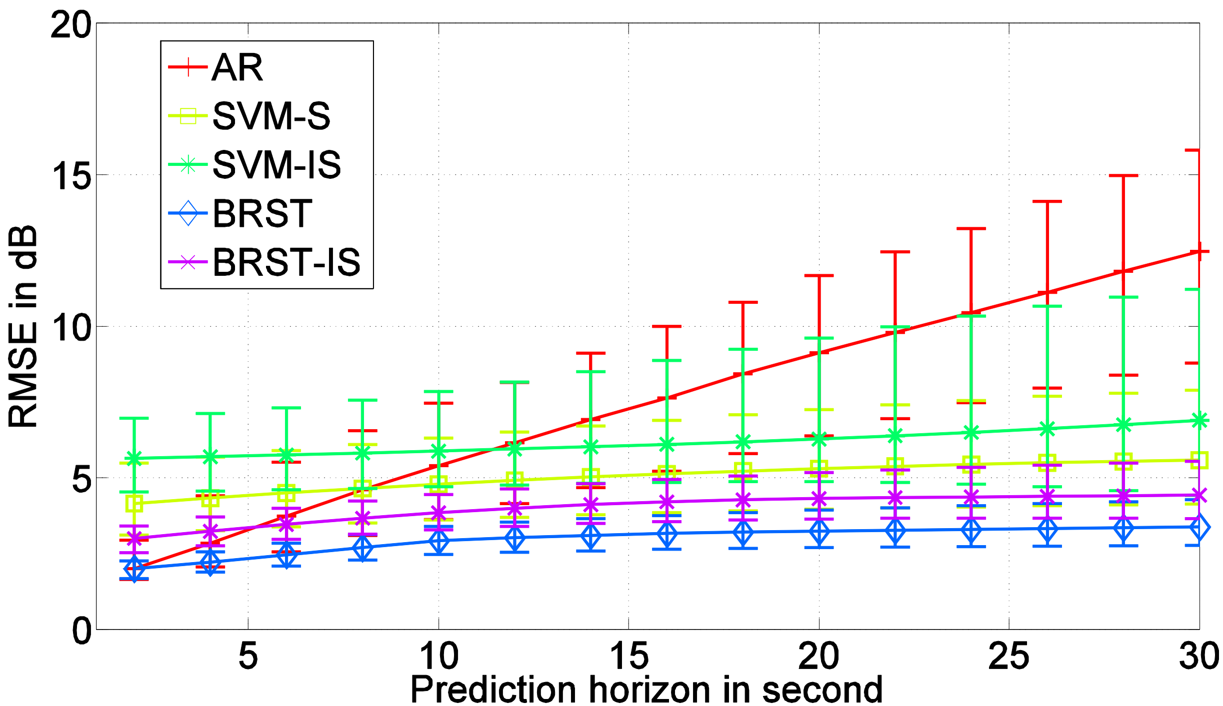
Empirical Evaluation

- Comparison between BRST(spatial and temporal), AR(temporal) and SVM(spatial)
  - BRST achieves significantly higher accuracy than AR
  - Exploiting spatial correlation in SVM and BRST improves the prediction accuracy for larger horizons
  - Comparing BRST and SVM with inaccurate location information (estimation error uniformly distributed within a radius of 150m) shows that BRST is more robust than SVM against inaccurate locatio



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Summary

- We proposed a general Bayesian framework for average channel prediction that jointly exploits temporal and spatial correlation.
- Based on the statistical properties of the Rayleigh fading channel, the predictive model is written as the combination of
  - A non-zero mean AR process, and
  - a zero mean GP.
- Bayesian inference is used to estimate prediction parameters and distributions.
- Numerical results for a realistic scenario show that
  - Our algorithm outperforms auto-regression for mid-and long-term prediction, and
  - It is substantially more robust to inaccurate localization than SVM.

Predictive model:

autoregressive process (temporal) + functional linear regression (spatial)

Reference

Liao, Q., Valentine, S. and Stanczak S., *“Channel gain prediction in wireless networks based on spatial-temporal correlation”*, The 16^{th} IEEE international workshop on signal processing advances in wireless communications (SPAWC), 2015

Kasparick, M., Calvalcante, R. L. G., Valentine, S., Stanczak, S. and Yukawa, W., *“Kernel-based adaptive online reconstruction of coverage maps with side information”, arXiv: 1404.0979*, 2014

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