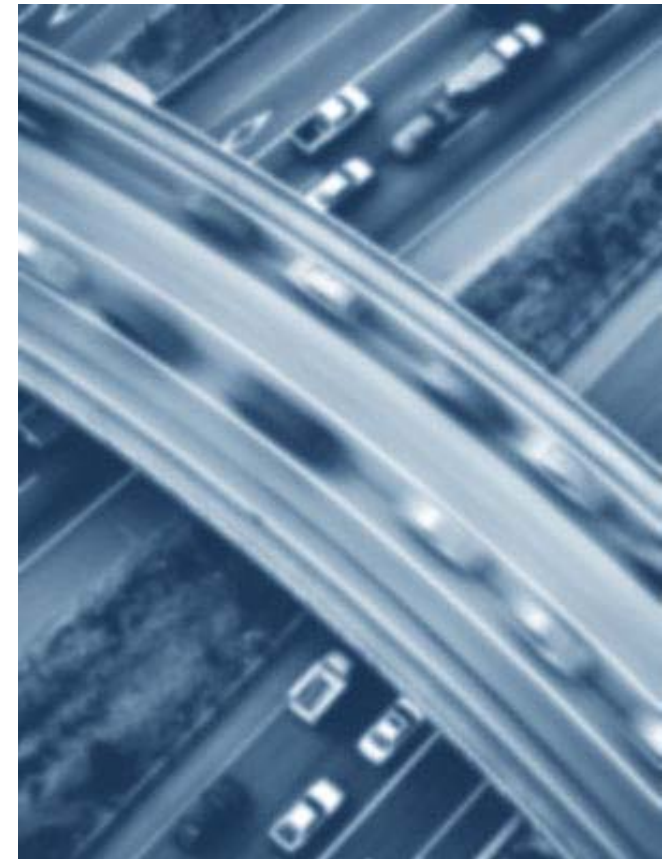


Possibilities of Mobility Modeling using Mathematical Methods

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Detecon International GmbH

Basierend auf Dissertation
„Beiträge zur Berechnung bewegungs-
abhängiger Kenngrößen von Mobilfunknetzen“



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Mobility Modeling – Why?

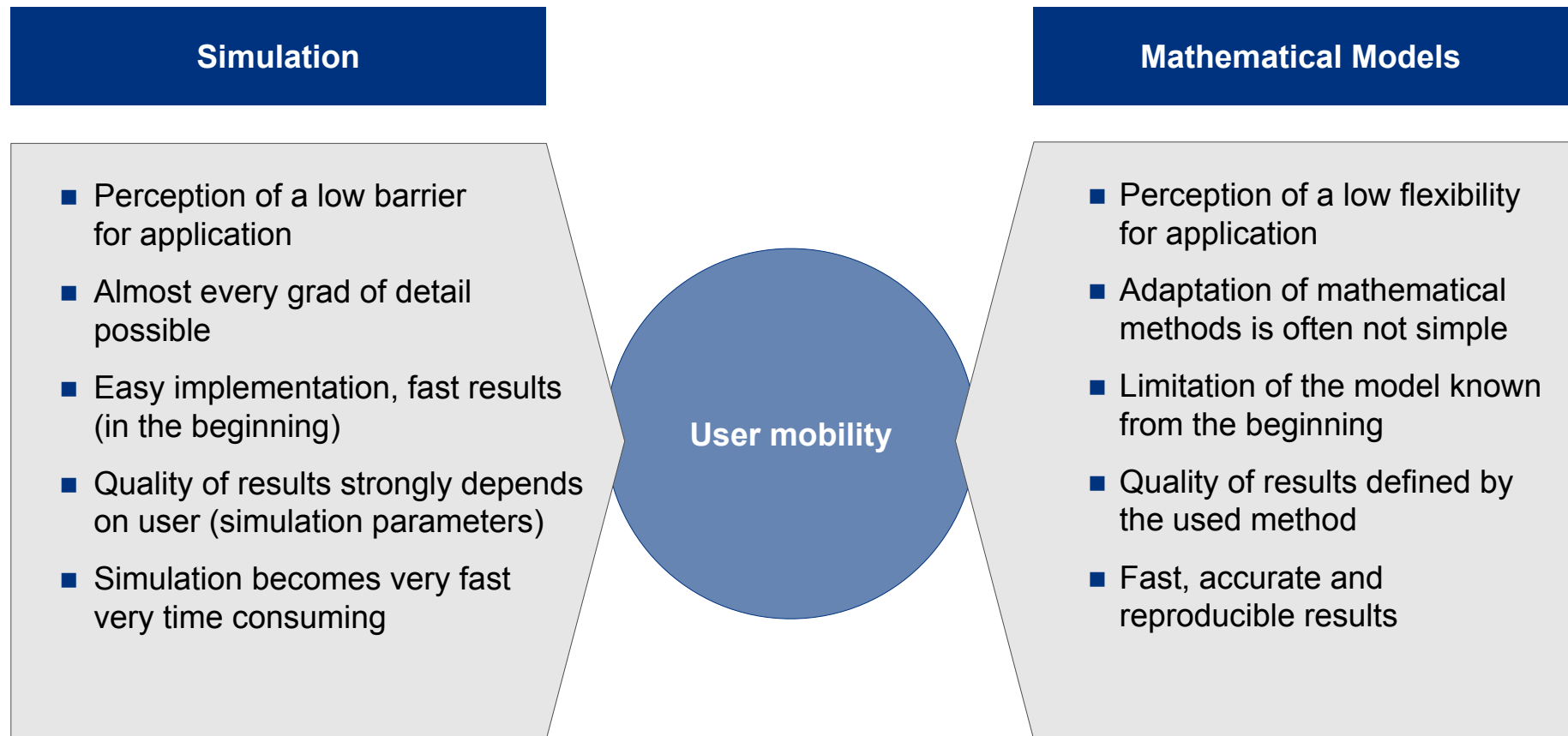
- Fixed Networks
 - User has to adopt his communication behavior to the network.
- Mobile Networks
 - The network has the ability to adapt to the behavior of the customer.
- Operation and planning of mobile networks reflects this adaptation
 - A lot of mobility-related functional units: HLR, VLR
 - Traffic caused by mobility of users: Management of the current location of customers, hand-over between access points to the network, signaling of movement
 - Sparse resources require efficient management of mobility related parameters
 - Mobility modeling necessary for development, operation, testing of the network

Challenges for Mobility Modeling

Everybody is „mobile“ → Sometimes too much information.

- It is almost impossible to model all mobility behaviors
 - Problem of selection of important behaviors
 - Problem of modeling these selected behaviors
- Mobility modeling is one input parameter for the simulation
 - Beside tasks like service modeling, resource allocation, queuing strategies, rate adaptations and others
- Selecting / Adjusting parameters
 - Measurement in operating network is difficult
 - Occupies resources (CPU for measurement, link capacities for collection, storage capacity, ...)
 - Security issues (storage device has access to all measurement points)
 - Often measurement of the first moment (mean)
 - Sometimes measurements of histograms (limited number of values) or higher order moments

Two Methods for Mobility Modeling



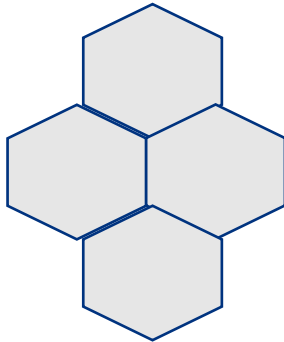
Input Parameters of Mobility Models

Service	„Traditional“ voice, SMS, MMS, data transfer, mail, ...
Duration	Day, hours, duration of the service usage, only during transmissions of packets, ...
Position	Everywhere within the investigated area, only in specific parts (streets) Movement starts always at the border of cells, anywhere within the cell (distribution)
Movement Direction	No changes of the direction, random changes, changes driven by topology (crossings of streets), changes occur regularly (every 50 m, every 20 sec), ...
Velocity of Movement	Constant, ruled by topology (Highway, slowdown in curves), randomly changed (when, how), stop during call, ...
Shape of Cells	Regular (Circle, hexagon, square, ...) Irregular like in real networks

Convergence Problem

The shape of cells in mobile communications systems as an example for the differences between the different worlds.

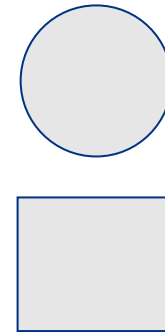
Simulation Model



Real Topology



Mathematical Model

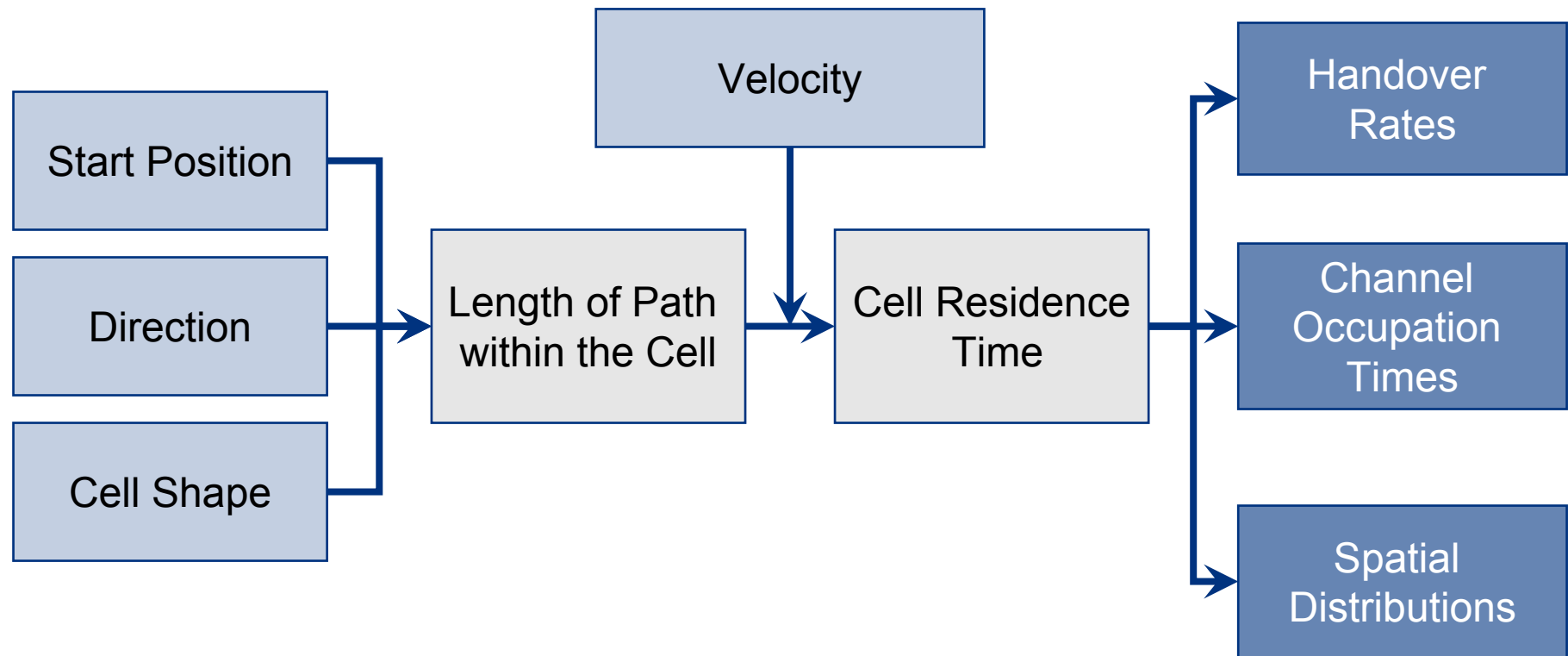


Main Problem: Convergence of the results

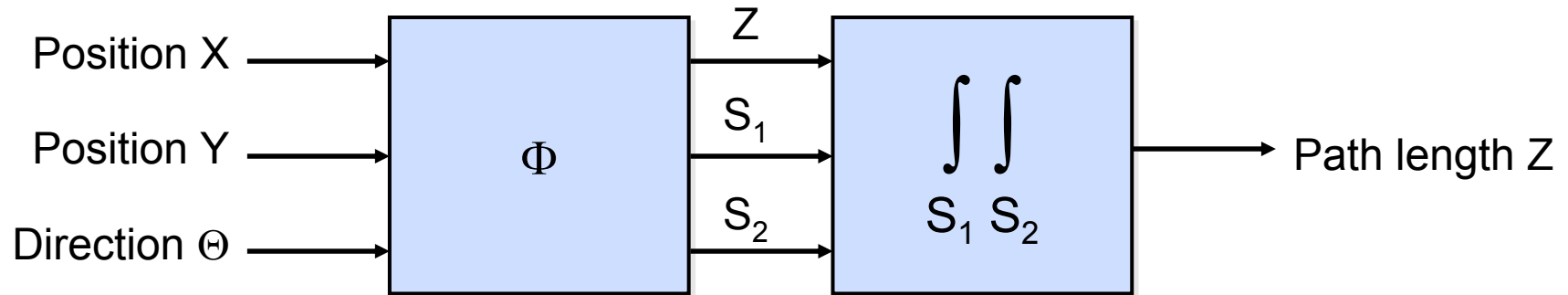
If there is no convergence, a decision based on simulation results is different than a decision based on results achieved by mathematical methods, and in the worst case both don't solve the problem in the real network

Calculation of Mobility Related Parameters

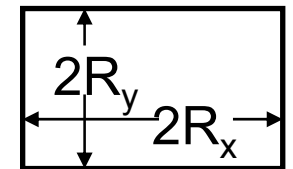
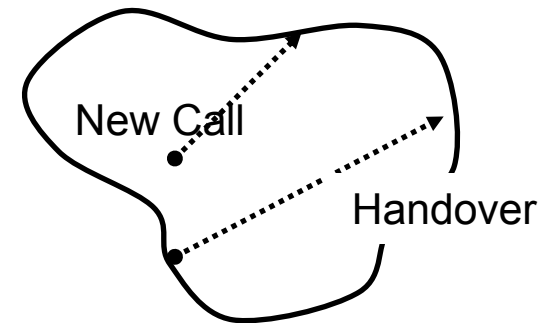
The transformation of probability density functions provides a mean for the derivation of the distribution of mobility related parameters.



Derivation of the Path Length in Rectangular Cells



- Two types of calls:
 - New Call: Starting anywhere within the cell
 - Handover: Starting at the border of the cell
- Used distribution for input parameters
 - Position: Equally distributed (within the cell for new calls, at the border of the cell for handover)
 - Movement direction: Equally distributed for new calls,
- Transformation Φ : several functions
 - Necessary to achieve the one-to-one relation of the transformation
 - Transformation valid for $\pi/2$ (quarter of a circle)

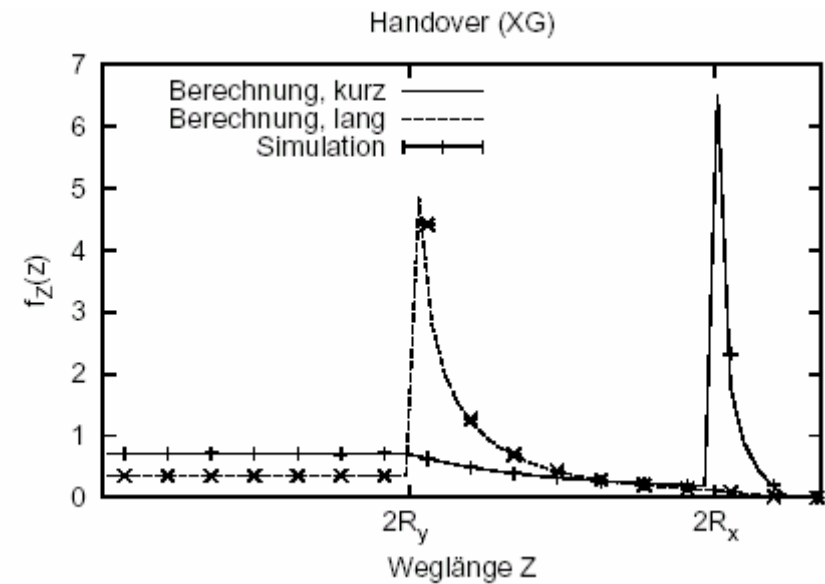
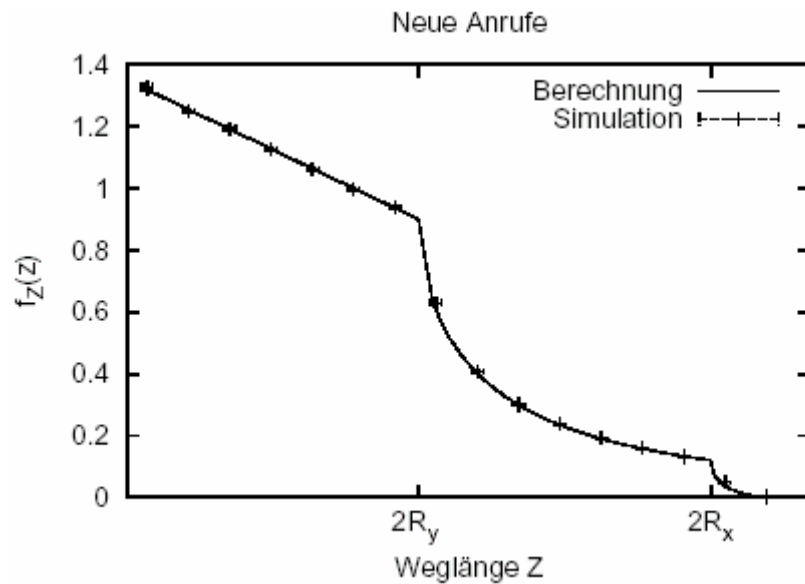


ITG

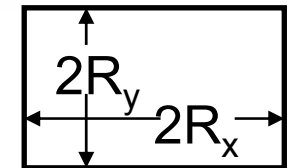
Path Length in Rectangular Cells

The functions reflect a main characteristics of the rectangle, the length of ist borders.

- Szenarios: - New call
- Handover at the short border
 - Handover at the long border



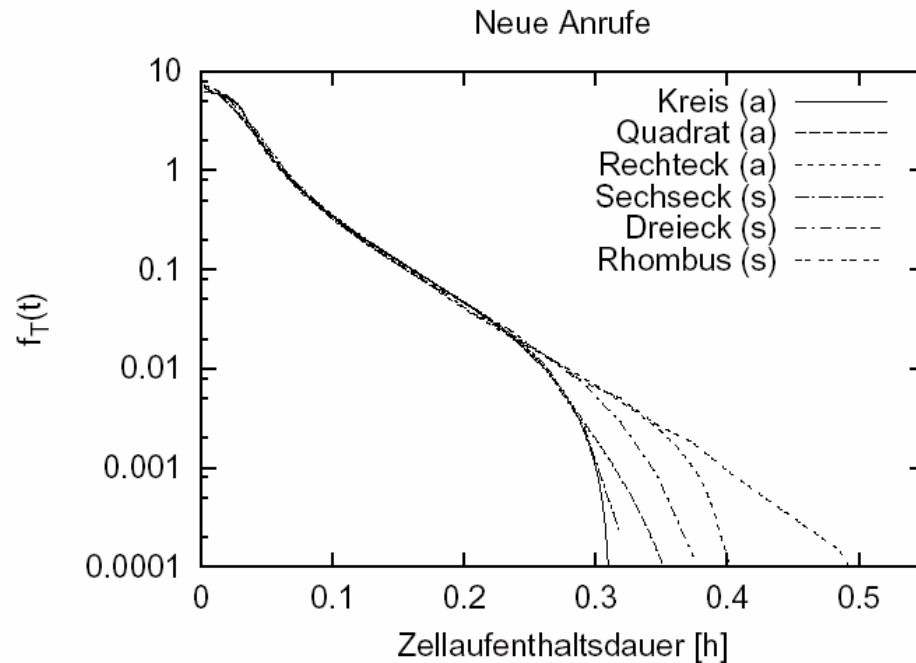
Parameter: Area $A=1\text{m}^2$; $R_y/R_x=0.5$; Simulation: 10 runs with 100 000 realizations each, confidence level 95%



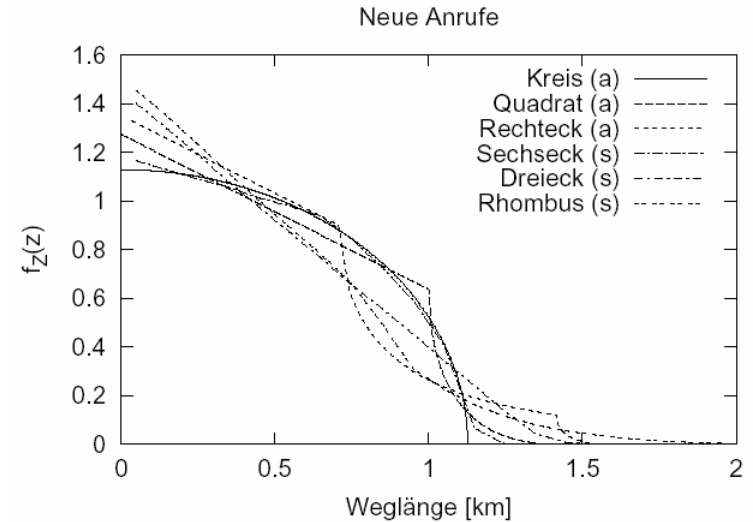
ITG

Comparison of Cell Residence Time for New Calls

The influence of the cell shape is very low. The usage of the circle as reference cell shape is correct.



Probability density function of path length:



Investigated cell shapes:

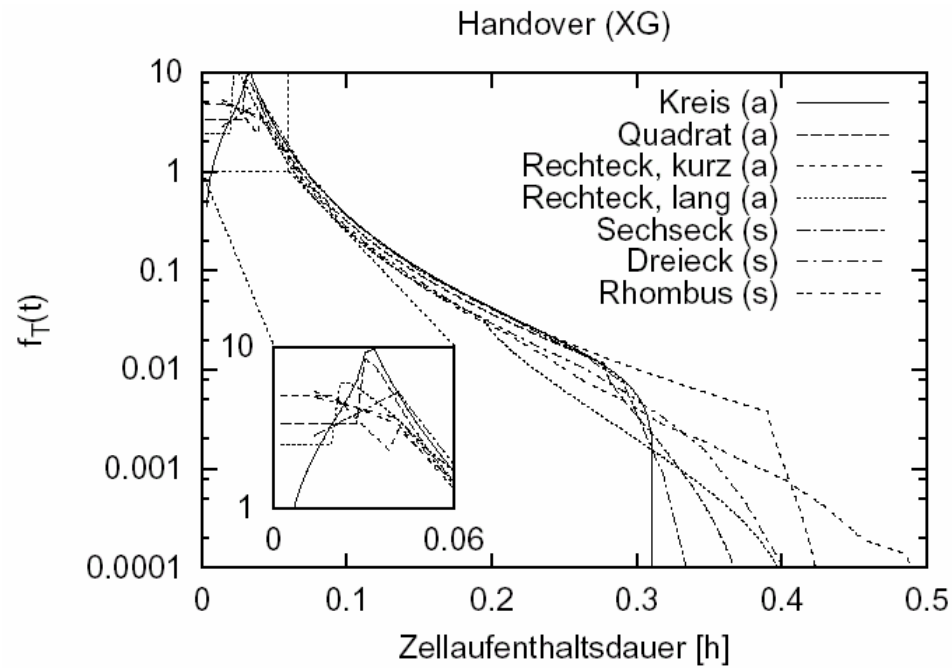


Input parameters:

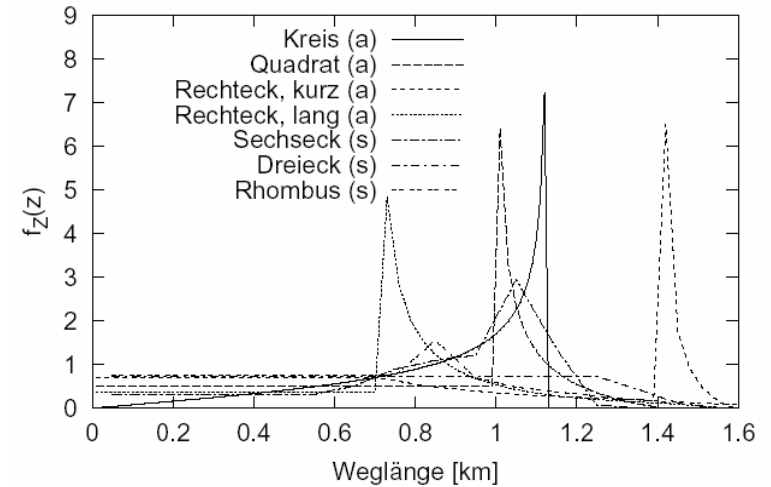
Area: constant = 1 km²
 Rectangle: $R_y/R_x = 0.5$
 Rhombus: $\alpha = 45^\circ$

Distribution of Cell Residence Time for Handover

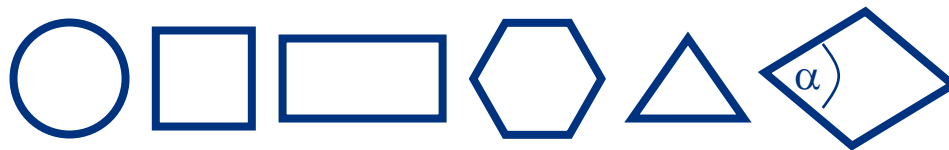
The influence of the cell shape is higher than for new calls.



Probability density function of path length:
Handover (XG)



Investigated cell shapes:



Input parameters:

Area: constant = 1 km²
 Rectangle: $R_y/R_x = 0.5$
 Rhombus: $\alpha = 45^\circ$

Error of Circular Approximation

The errors regarding mean path length and mean cell residence time of the approximation of a randomly shaped cell are equal.

Error of the Approximation of a cell shape by an circle:

$$e = \frac{E[\cdot]_{\text{Circle}} - E[\cdot]_{\text{Cell}}}{E[\cdot]_{\text{Cell}}} = \frac{E[\cdot]_{\text{Circle}}}{E[\cdot]_{\text{Cell}}} - 1$$

The relation between 2 first moments of path length and cell residence time:

$$\frac{E[T_1]}{E[T_2]} = \frac{E[Z_1] \int f_V(v)/v \, dv}{E[Z_2] \int f_V(v)/v \, dv} = \frac{E[Z_1]}{E[Z_2]}$$

For the comparison of different cell shapes a common system is required

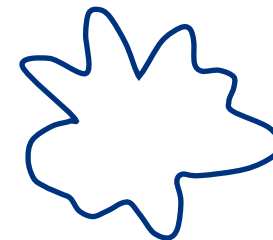


Circle Equivalence

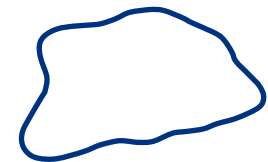
Circle Equivalence (Kreisähnlichkeit) ceq

- Enables comparison of different cell shapes
- Input parameters Area A and circumference u available for any cell shape
- Limited between 0 and 1
 - The circle has the smallest circumference for a given area






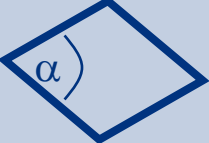
$$ceq \equiv 4\pi \frac{A}{u^2} \quad 0 < ceq \leq 1$$



$ceq = 0.4$



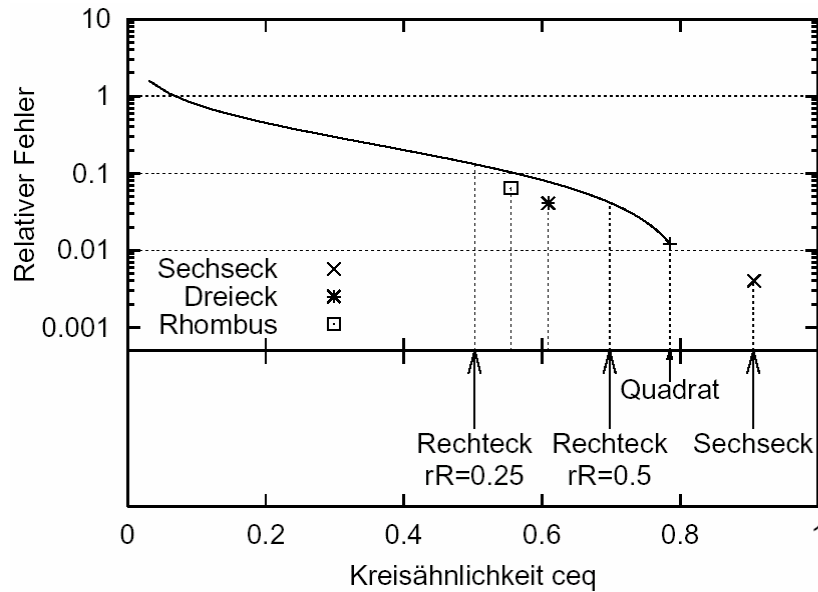
$ceq = 0.8$

Cell	Circle	Hexagon	Square	Equilateral Triangle	Rectangle	Rhombus
						
ceq	1	0.907	0.785	0.609	$\frac{\pi r_R}{(1+r_R)^2}$	$\frac{\pi}{4} \sin(\alpha)$

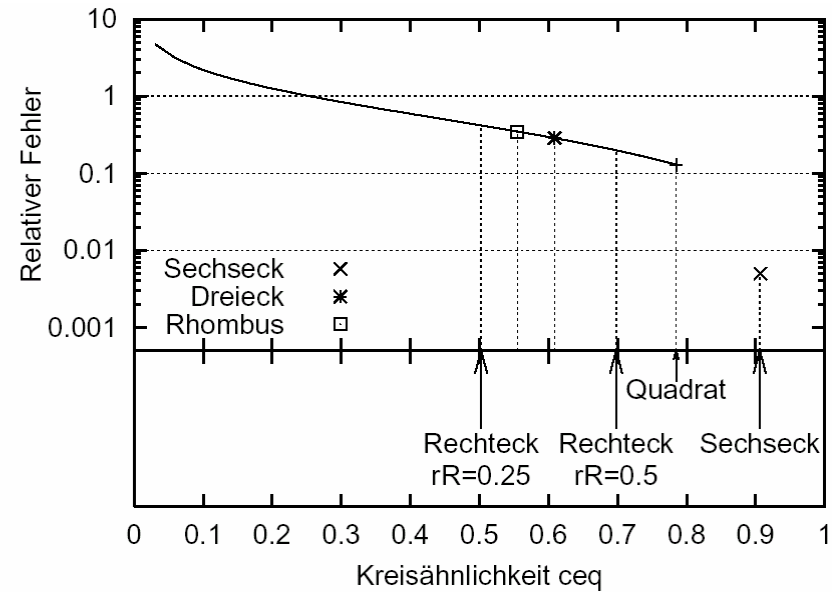
Circular Approximation

The rectangular approximation of a given cell can be used to estimate the error of the circular approximation of the same cell.

New Call



Handover

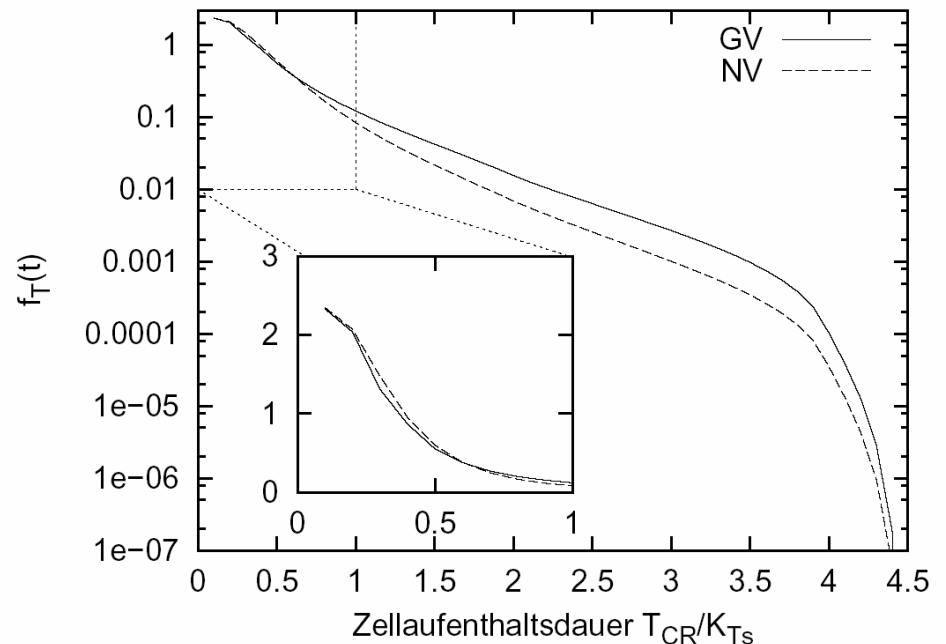


Influence of the Distribution Type of the Velocity

The type of the distribution of the velocity has influences the cell residence time. If this has to be considered, depends on the scenario.

Investigation Parameters:

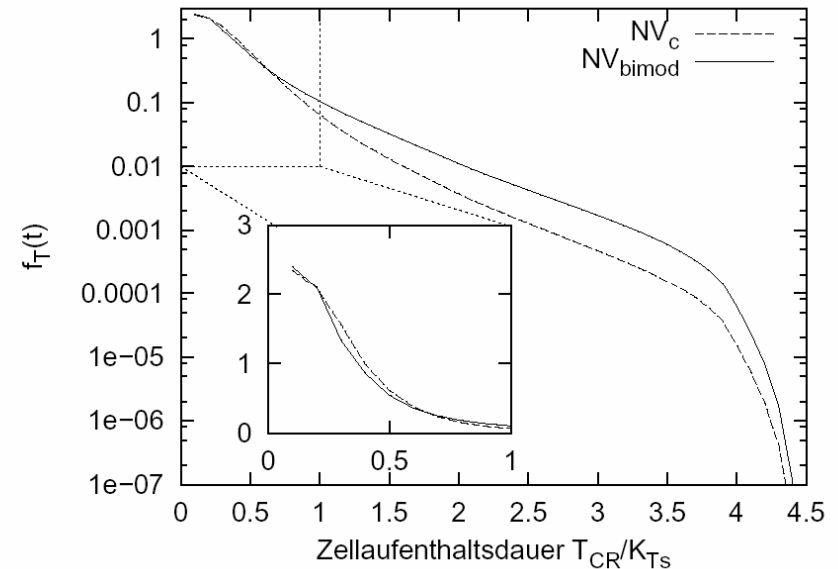
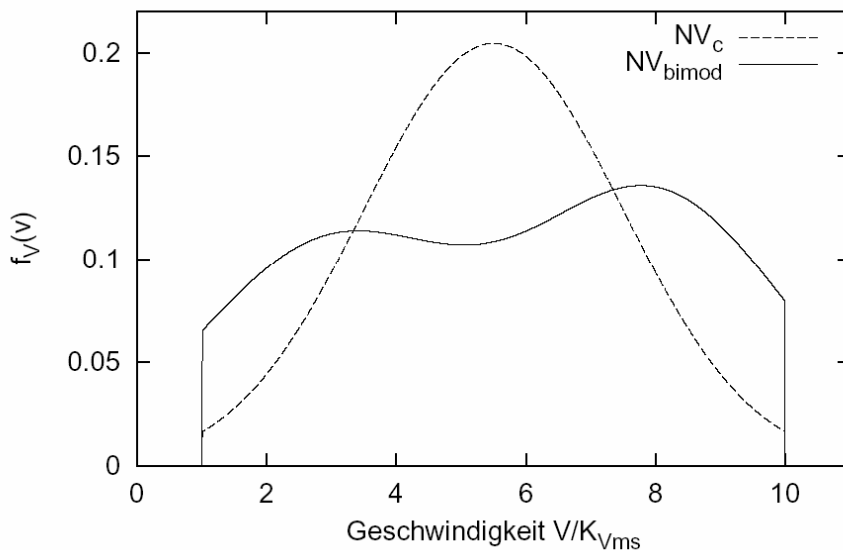
- Investigation for
 - Normal distribution (NV)
 - Equal distribution (GV)
- Both distributions are limited to: $V_u \leq v \leq V_o$
 - Speed in our world is limited
 $0 \leq v \leq 200\,000 \text{ km/s}$
 - Especially $v=0 \text{ m/s}$ causes some “trouble” → unlimited cell residence time (this is theoretically, but not practically possible)



Scenario: rectangle, new calls, $R_x=2R_y=1\text{m}$, $V_u=V_o/10=1\text{m/s}$

Multimodal Distribution of the Velocity

The type of the distribution of the velocity is not limited to simple scenarios!



Scenario: rectangle, new calls, $R_x=2R_y=1\text{m}$, $V_u=V_o/10=1\text{m/s}$, $K_{Vms}=1\text{m/s}$, $K_{Ts}=1\text{s}$

- Bimodal distribution of velocity
 - Two normal distributions
 - Comparison to a single normal distribution
 - Both limited between V_u and V_o

- Parameters
 - NV_c : $E[V]=5.5$, $VAR_1[V]=4 \text{ m}^2/\text{s}^2$
 - NV_{bimod} :
 $E_1[V]=5.5$, $VAR_1[V]=4 \text{ m}^2/\text{s}^2$, $p_1=0.45$
 $E_2[V]=8$, $VAR_2[V]=4 \text{ m}^2/\text{s}^2$, $p_2=0.55$

Conclusion and Outlook

■ Past

- Circle, Equal Distribution

■ Present

- Circle, Rectangle (Square)
- Influence of distribution types of input values (e.g. velocity)
 - Equal distribution, normal distribution, multimodal distributions
- Models for spatial distribution

■ Future

- **...is mobile!**
There are definitely applications for mathematical models of user mobility.