When 10 dB doesn't equal 10 dB: Performance Prediction for Future Cellular Networks

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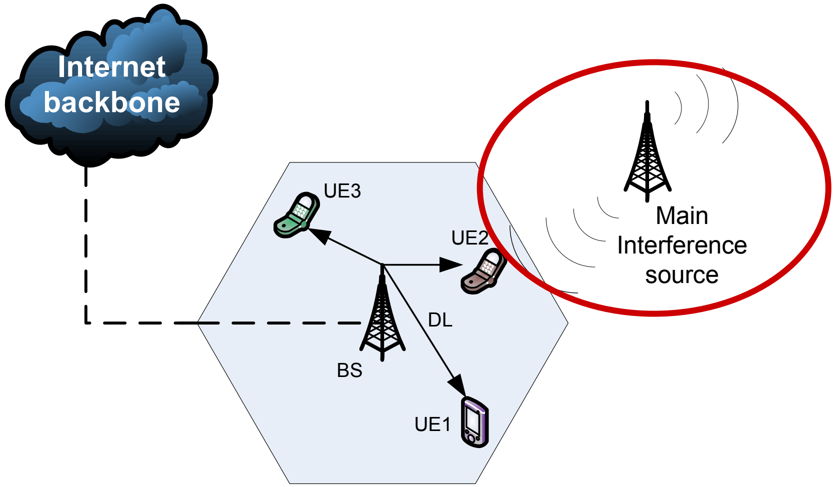
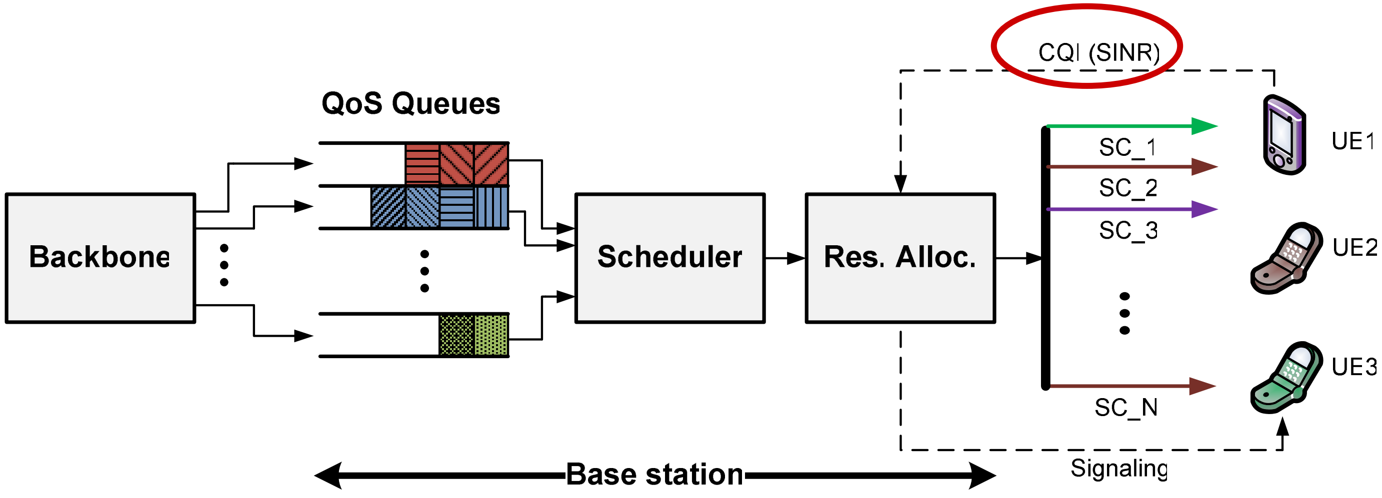
Outline

- Introduction & System Model
- Problem statement
- OFDMA channel transformations
- System application
- Conclusions

System Model



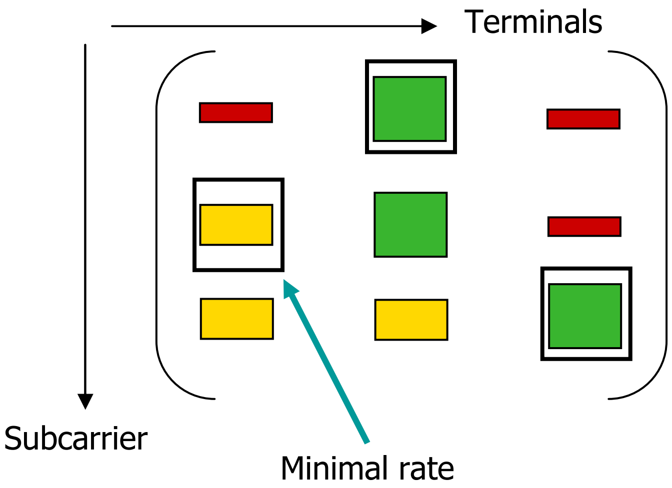
- **OFDM-based centralized system (e.g. WiMAX, LTE)**
- **Various traffic types with different QoS requirements**
- **For each downlink phase, a scheduler passes J packets to the resource allocation unit**



System Model II

- **Optimize performance by assigning subcarriers dynamically**
  - Requires channel knowledge & adequate time coherence
  - Known to outperform static schemes [Wong99, Rhee00]
- **How to assign subcarriers optimally?**
  - Maximize minimal rate ➔ rate adaptive approach [Ergen03]

Subcarrier states



$$\begin{aligned} \max \quad & \epsilon \\ \text{s. t.} \quad & \sum_j x_{j,n} \le 1 \quad \forall\, n \\ & \sum_n b_{j,n} \cdot x_{j,n} \ge \alpha_j \cdot \epsilon \quad \forall\, j \end{aligned}$$

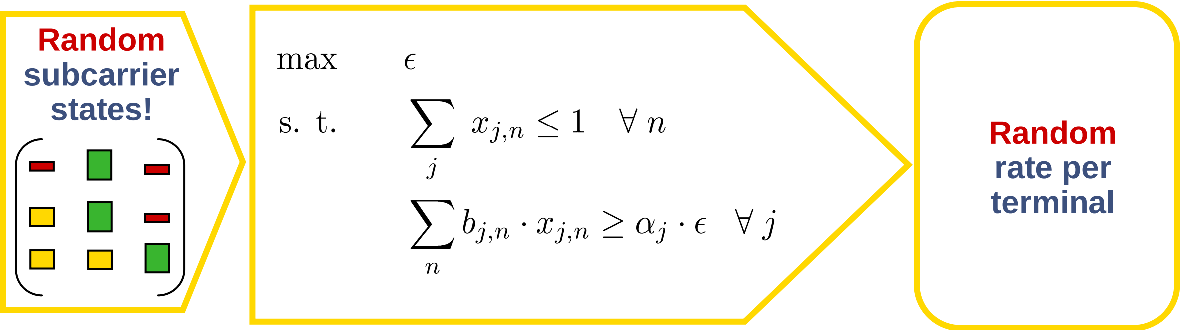
$x_{j,n}$: Binary assignment variable

$b_{j,n}$: Capacity per subcarrier/terminal pair

α_j : Packet weight factor

Problem Statement

- **Assume a rate-adaptive scheme to be in place**
- **Scheduler basically needs some notion of ϵ**
  - Should depend on J !
- **Why is this difficult?**
  - ϵ is generated by adaptive algorithm
  - ϵ is essentially a random variable!



Random subcarrier states!

$$\begin{aligned} \max \quad & \epsilon \\ \text{s. t.} \quad & \sum_j x_{j,n} \leq 1 \quad \forall\, n \\ & \sum_n b_{j,n} \cdot x_{j,n} \geq \alpha_j \cdot \epsilon \quad \forall\, j \end{aligned}$$

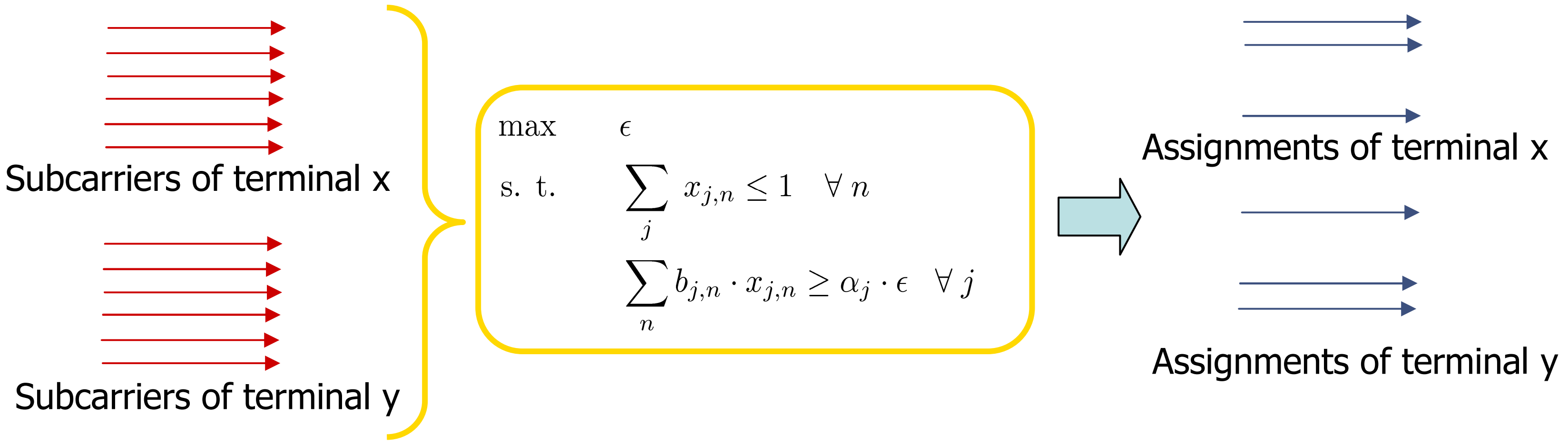
Random rate per terminal

➔ **Any scheduling decision is related to an outage probability**

- Outage: Scheduled data unit can not be transmitted during DL

OFDMA Channel Transformation

- **How to obtain a PDF of ϵ ?**
- **ϵ relies on channel gain statistics of <u>assigned subcarriers</u>**



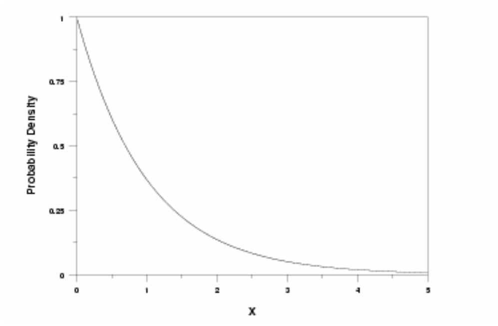
Subcarriers of terminal x

Subcarriers of terminal y

$$
\begin{aligned}
\max \quad & \epsilon \\
\text{s. t.} \quad & \sum_j x_{j,n} \leq 1 \quad \forall\, n \\
& \sum_n b_{j,n} \cdot x_{j,n} \geq \alpha_j \cdot \epsilon \quad \forall\, j
\end{aligned}
$$

Assignments of terminal x

Assignments of terminal y



$\xrightarrow{\textbf{OFDMA channel transformations!}}$

?

Interference-Limited Channel Transformations I

- **Exponentially distributed signal and interference gains:**

$$g_j^s \sim \mathrm{Exp}(\tfrac{1}{\rho_j^s})$$

$$g_j^i \sim \mathrm{Exp}(\tfrac{1}{\rho_j^i})$$

- **Fixing the transmit and interference power yields the SINR:**

$$\gamma_{j,n} = \frac{p_n^s \cdot g_{j,n}^s}{p_n^i \cdot g_{j,n}^i + \sigma_n^2}$$

- **PDF** $f_{\gamma_{j,n}}(y) = \left[\frac{\sigma^2}{P_I y + P_S} + \frac{P_I P_S}{(P_I y + P_S)^2}\right] \cdot e^{-\frac{\sigma^2}{P_S} y}$

- **CDF** $F_{\gamma_{j,n}}(y) = 1 - \frac{P_S}{P_I y + P_S} \cdot e^{-\frac{\sigma^2}{P_S} y}$

where $P_S = p_n^s \cdot \rho_j^s$ **and** $P_I = p_n^i \cdot \rho_j^i$**.**

Interference-Limited Channel Transformations II

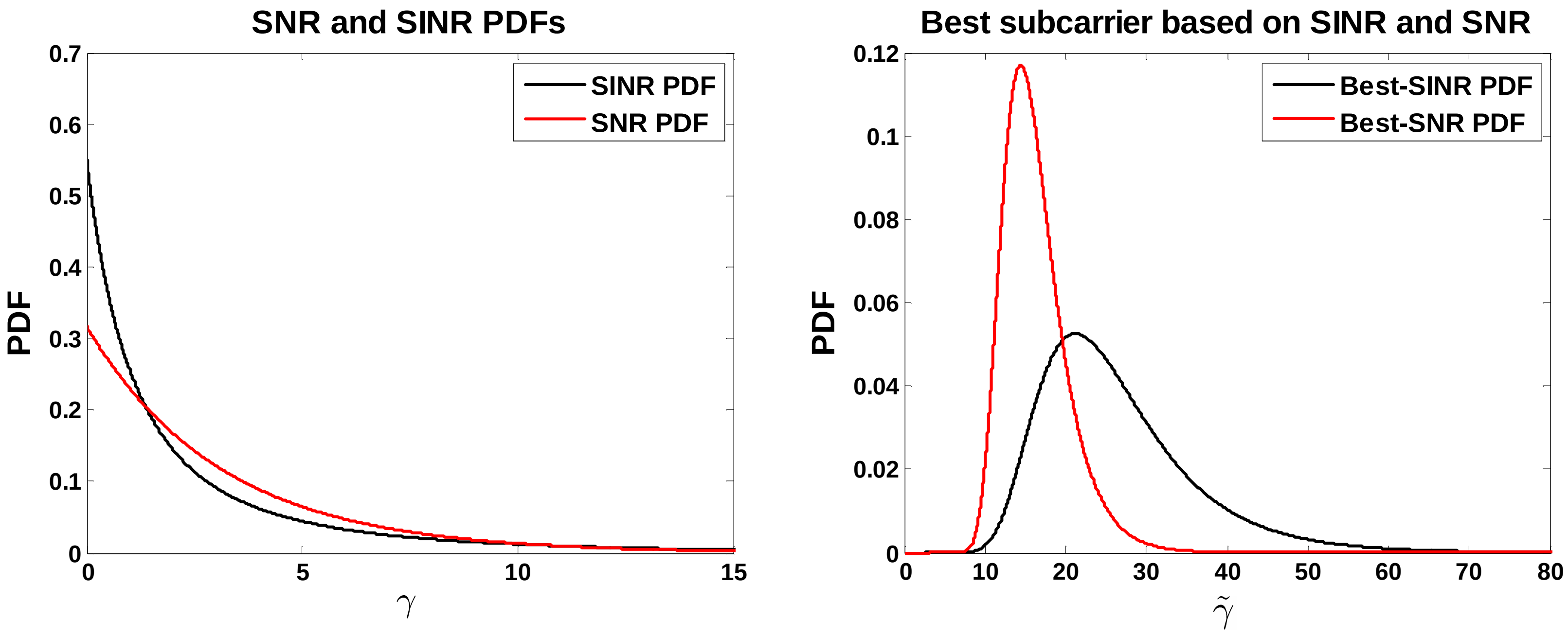
- **Derivation of exact statistics is difficult (impossible?):**
  - Exhaustive search (NP hard)!
- **Analyze a suboptimal algorithm ➔ approximate optimum**
  - Can be done by applying order statistics
- **Example resulting PDF and CDF of the best subcarrier:**

$$\begin{aligned} f_{\tilde{\gamma}_{j,(1)}}(x) &= A_{j,(1)} \left[1 - \tfrac{P_S}{P_I x + P_S} \cdot e^{-\frac{\sigma^2}{P_S} x}\right]^{A_{j,(1)}-1} \cdot \\ &\left[\tfrac{\sigma^2}{P_I x + P_S} + \tfrac{P_I P_S}{(P_I x + P_S)^2}\right] \cdot e^{-\frac{\sigma^2}{P_S} x} \end{aligned}$$

$$F_{\tilde{\gamma}_{j,(1)}}(x) = \left[1 - \frac{P_S}{P_I x + P_S} \cdot e^{-\frac{\sigma^2}{P_S} x}\right]^{A_{j,(1)}}$$

Illustration of SINR PDFs

- 48 subcarrier, 6 terminals (only first terminal considered)
- Noise-limited system vs. interference-limited system
- Average SNR/SINR = 5 dB



Higher gains in the interference-limited case!

System Application: Rate PMFs

- **Given a system specification:**
  - Adaptive modulation with SNR/SINR switching points
  - Rate per subcarrier is a random variable
  - ➔ Obtain rate PMFs - $z_{j,(i)}$ - for each chosen subcarrier based on SNR/SINR distribution functions

- **Total rate per terminal is sum of single subcarrier rates:**

$$Z_j = \sum_{i=1}^{l_j} z_{j,(i)}$$

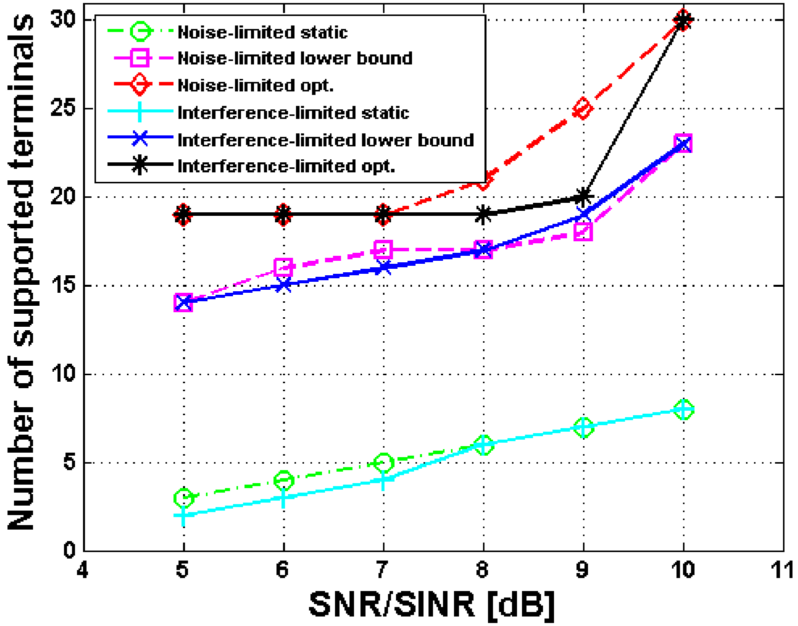
- **Total rate PMF is obtained by convolution:**

$$p\left(Z_j\right) = \bigodot_{i=1}^{l_j} p\left(z_{j,(i)}\right)$$

Note: <u>This is only true if random variables are independent</u>.
This is not the case (order statistics!), we still apply this as approximation and compare the obtained bound with simulations!

Numerical Results I

- **VoIP capacity for different SNR / SINR settings**
  - Comparison schemes:
    - Static resource allocation (diversity schemes, no CQI usage)
    - Dynamic (optimal) allocation (Band AMC, simulated performance)
    - Bound on optimal allocation

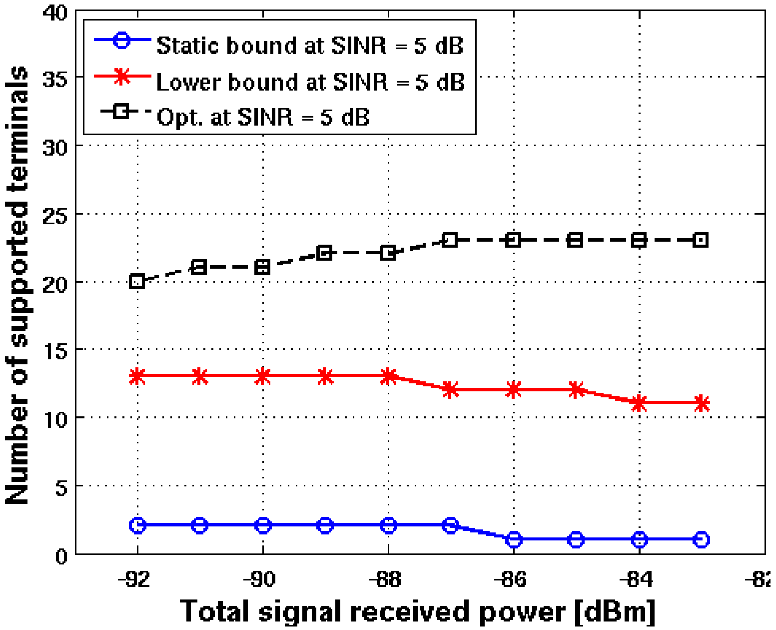


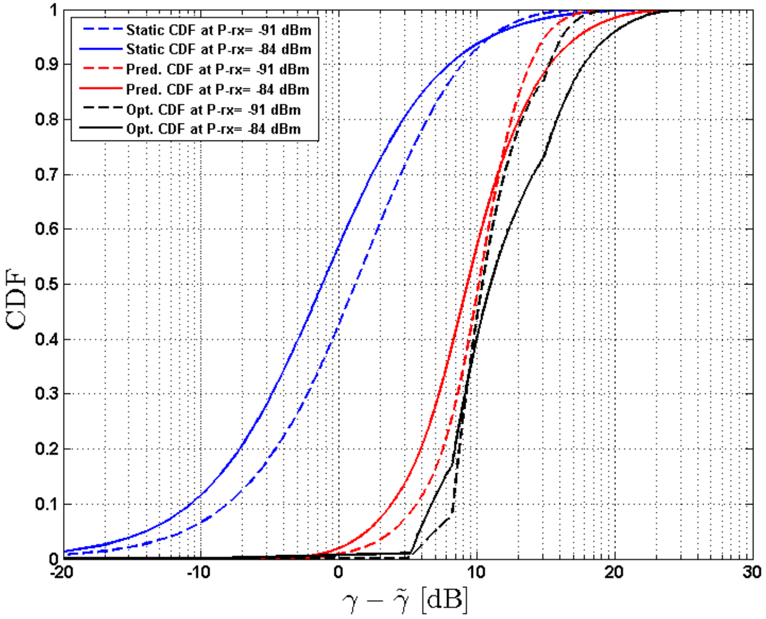
Parameters:
- **10 MHz bandwidth**
- **96 subchannels**
- **4 modulation types (BPSK, QPSK, 16 QAM 64 QAM)**
- **5 ms frame length**
- **S = 24 symbols**
- **Convolutional coding rate ¾**
- **Required BER: 0.0052**
- **VoIP outage prob: 0.05**

- In the interference-limited case: received signal power is fixed while interference power is increased (starting at 10 dB)

Numerical Results II

- **Effect of received powers on system capacity:**
  - Fixed average SINR 5 dB
  - Varying received signal power and interference power



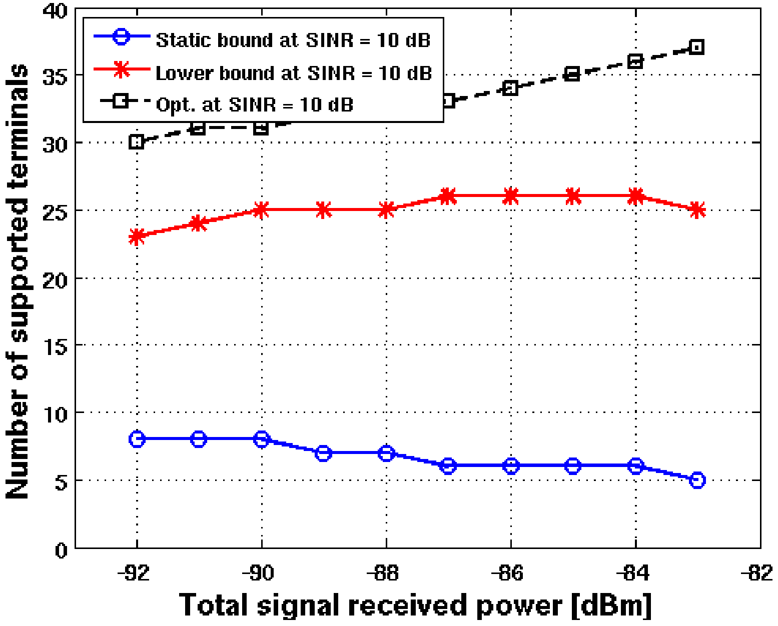


  - Diversity scheme has decreasing performance
  - Dynamic scheme has increasing performance !
  - Reason given by the SINR distributions

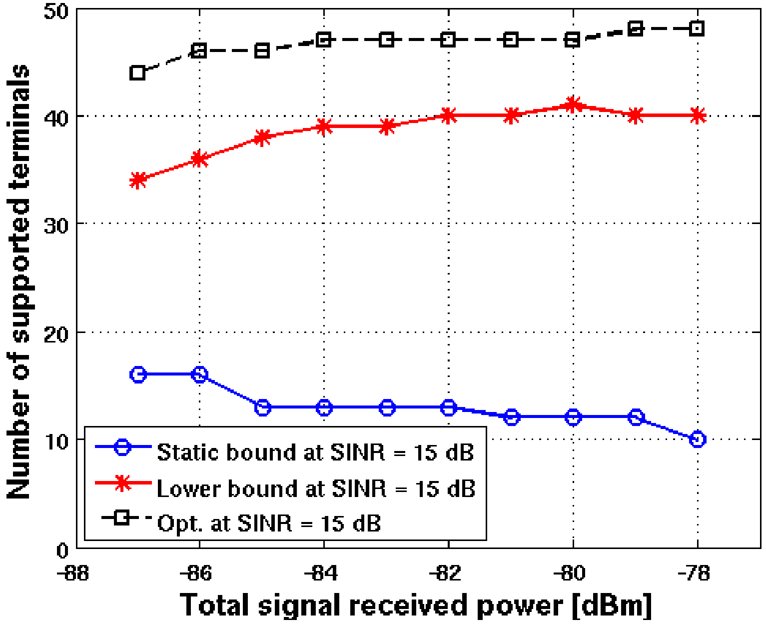
Numerical Results III

- Effect of different receive powers on VoIP capacity

10 dB



15 dB



- Same performance behavior observed as for the 5 dB case
- Performance prediction works well

Conclusions

- **Accurate performance models required for adaptive wireless networks:**
  - Admission control
  - Scheduling
  - Handoff decisions
  - Network planning

- **Difficult to obtain such models due to random behavior of the instantaneous capacity in dynamic algorithms**

- **This talk: models for interference-limited dynamic OFDMA**
  - Performance prediction possible, significant improvement of state-of-the art
  - Still, performance gap remains (recall: exhaustive search!)
  - Model reveals important performance characteristic for interference-limited OFDMA cells:
    - Using multi-user diversity: Don‘t care about interference, higher receive power is better (at constant SINR)
    - Diversity schemes: The lower the interference power the better is the performance (at constant SINR)
    - Reason due to SINR distributions and the influence of transmit and interference powers