Divide-and-conquer and signal scale alignment in relay networks

Aydin Sezgin

Emmy-Noether Research Group on Wireless Networks

In collaboration with:

S. Avestimehr (Cornell) , A. Khanjenejad (Caltech), B. Hassibi (Caltech), D. Tse (UC Berkeley)

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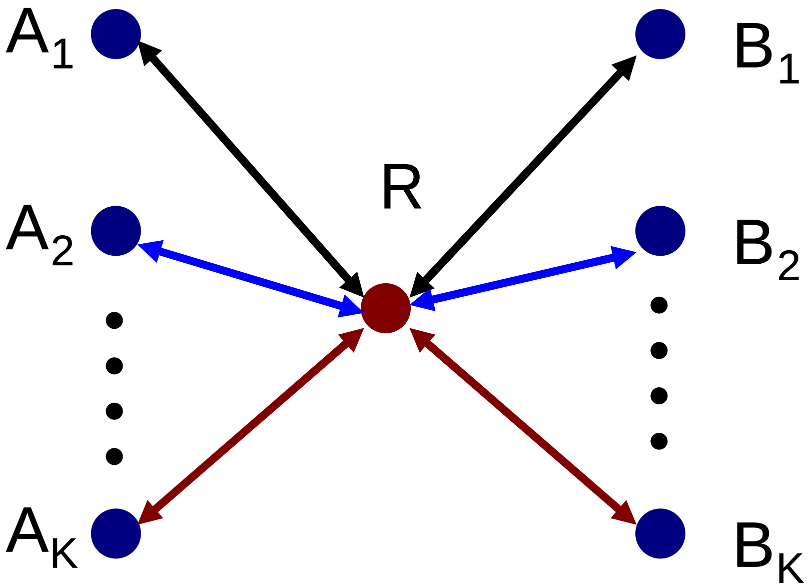
Agenda

- Multi-Pair Two-Way Relay Network: Setup
- Motivation
- Deterministic approach
- Results:
  - Characterization of the capacity region of the deterministic multi-pair two-way relay network
  - Approximation of the capacity region of the Gaussian two-pair two-way relay network
- Summary

Multi-Pair Relay Network

- Multiple Pairs want to communicate via a relay
- How to relay the information?
- Capacity region?



Motivation

- Why is the problem interesting?
  - Increasing demand for higher data rates in wireless networks
  - Today's systems not able to meet the demands
  - However: Increasing capability of nodes, new topologies
- Why is the problem hard?
  - Capacity of wireless networks: Holy grail of information theory
  - 60 years after Shannon we are still far away

Motivation

- How to make progress?
  - Instead of characterizing the capacity region exactly, only approximate up to a finite, small **gap**
- New strategies and ideas
  - **Codes with Structure**
    - Will they win the battle now?
  - Generalized degrees of freedom
    - Reveals quality of upper bounds and achievable schemes
  - **Interference alignment**
    - Sophistacted interference reduction
  - **Deterministic Approach**
    - Provides insights into behavior of Gaussian channels

Interference alignment

- Alignment exploiting/in
  - Delays
    - Interesting, but not realistic (so far) for practical systems
  - Time-varying channels
    - Interesting, but non-causal information needed
  - Frequency selective channels
  - Signal space
    - Useful for high SNR (around 20 dB)
  - **Signal level/scale**

- Deterministic Approach

Deterministic Channel Model

Gaussian Channel Model $\longrightarrow$ Deterministic Channel Model

$$y = \sum_{i}^{N} \sqrt{\mathsf{SNR}_i}\, x_i + z \quad \longrightarrow \quad \mathbf{y} = \sum_{i}^{N} \mathbf{S}^{q-n_i} \mathbf{x}_i \ (mod2)$$

S is the q x q shift matrix:

$$n_i = \lceil \log_2 (\mathsf{SNR}_i) \rceil$$

$$q = \max(n_i)$$

$$\mathbf{S} = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ \vdots & \ldots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & 1 & 0 \end{bmatrix}$$

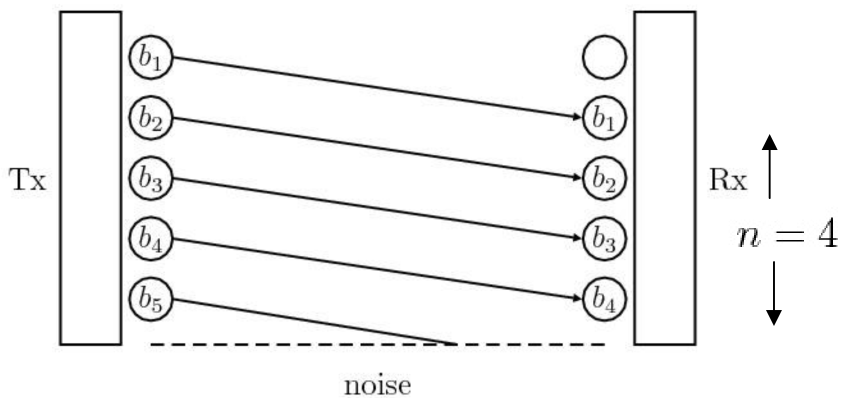
$\mathbf{x}_i$: Binary representation of the transmit signal

Bits transmitted at different signal level, MSB at highest level, LSB at lowest level

Depending on the SNR, a number of least significant bits are truncated at noise level

Deterministic channel model

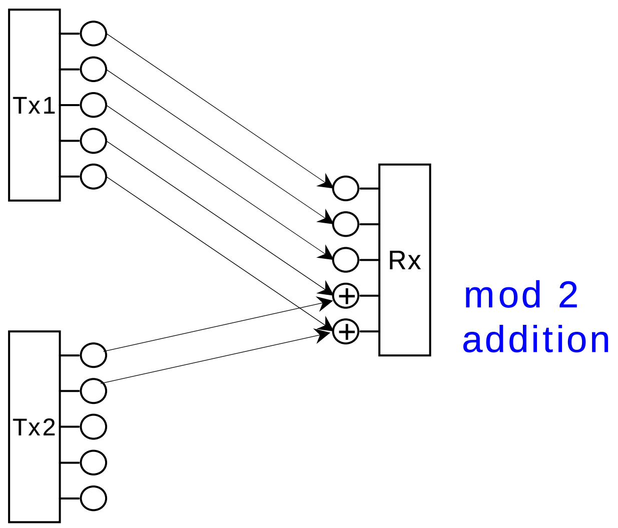
point to point



Receiver gets those bits that arrive above noise level

$$n = \lceil \log SNR \rceil$$

MAC



Receiver gets the modulo sum of those bits that arrive at the same signal level

Strategy

- From Gaussian to Deterministic and back

Gaussian

- Noise
- Fading

Deterministic

- Shifting

- Capacity C

Lower Bound on C= $\underline{C}$ - κ

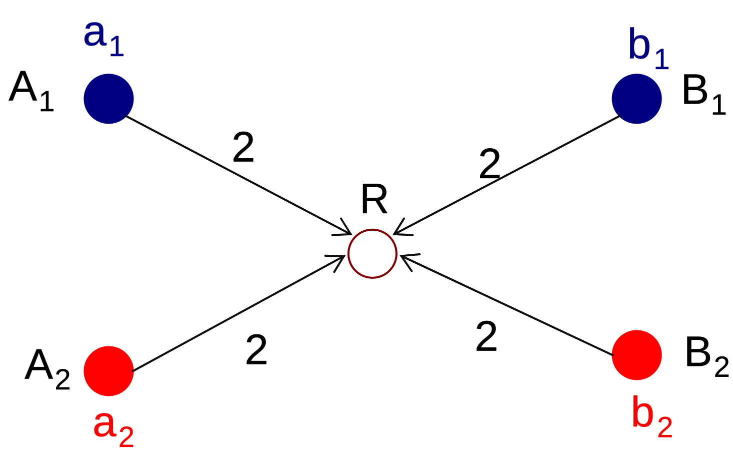
$\underline{C}$ –cut set bound

κ - constant, independent of SNR and channel gains

Deterministic case

- Simple transmission results in **random mixing** at relay
  => Does not seem to work
- We need to manage interference and carefully create and forward the superposed bits
- Idea: orthogonalize the pairs over signal levels (**Divide-and-conquer**)

- How to allocate the signal levels?

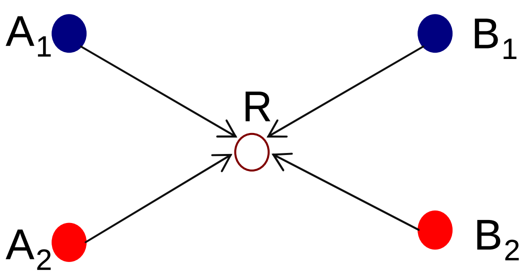


$$\alpha_{11}a_1+\alpha_{12}b_1\cancel{+\alpha_{13}a_2+\alpha_{14}b_2}$$

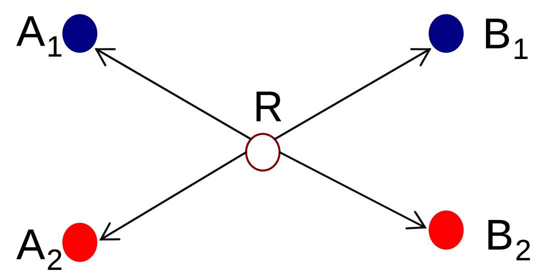
$$\cancel{\alpha_{21}a_1+\alpha_{22}b_1+}\ \alpha_{23}a_2+\alpha_{24}b_2$$

Structure of scheme (deterministic)

- Proposed scheme to achieve $(R_{A1}, R_{B1}, R_{A2}, R_{B2})$, $R_{Ai} > R_{Bi}$:
  - **Signal level alignment**: Create and forwards R_{Bi} superposed bits for pair i
  - Forward $R_{Ai}-R_{Bi}$ bits from A_i to B_i
  - Signal level allocation is according to the channel gains

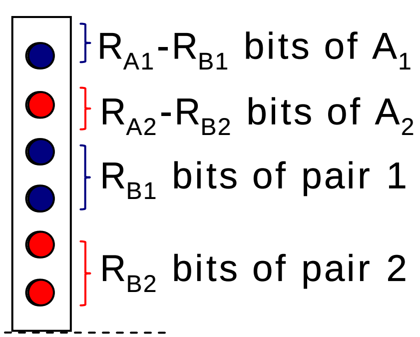


$$n_{A1R} \geq n_{A2R} \geq n_{B1R} \geq n_{B2R}$$

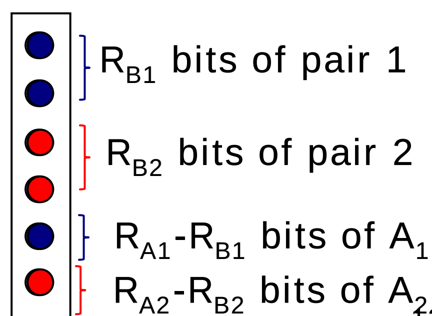


$$n_{RB2} \geq n_{RB1} \geq n_{RA2} \geq n_{RA1}$$

Received signal levels at the relay



Transmit signal levels at the relay



Result I: Deterministic Case

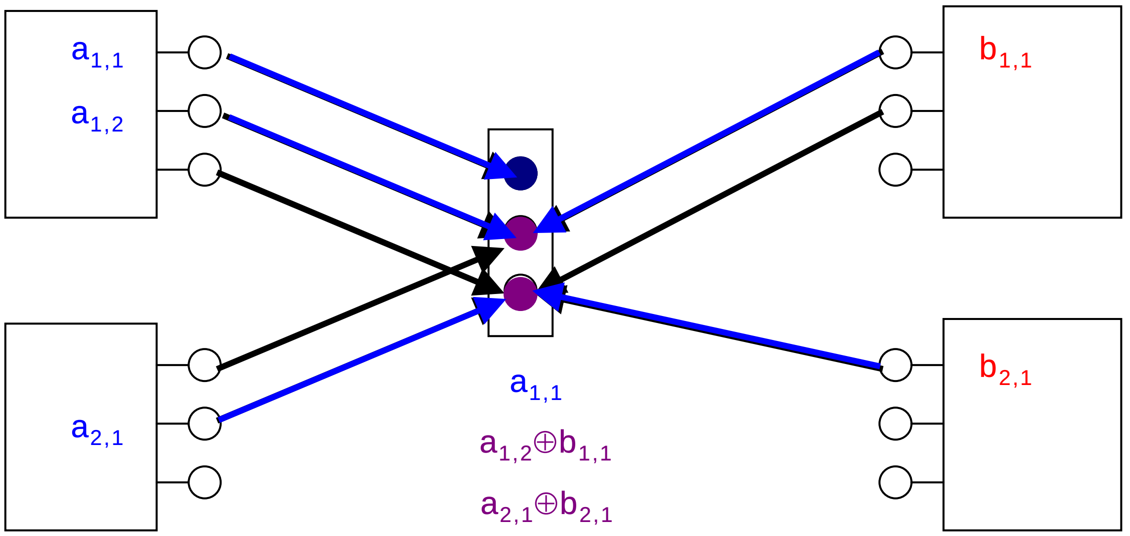
Deterministic multi-pair two-way relay network:

- Capacity region is equal to the cut-set upper bound region
- Cut-set upper bound is achieved by a divide-and-conquer strategy and signal level alignment

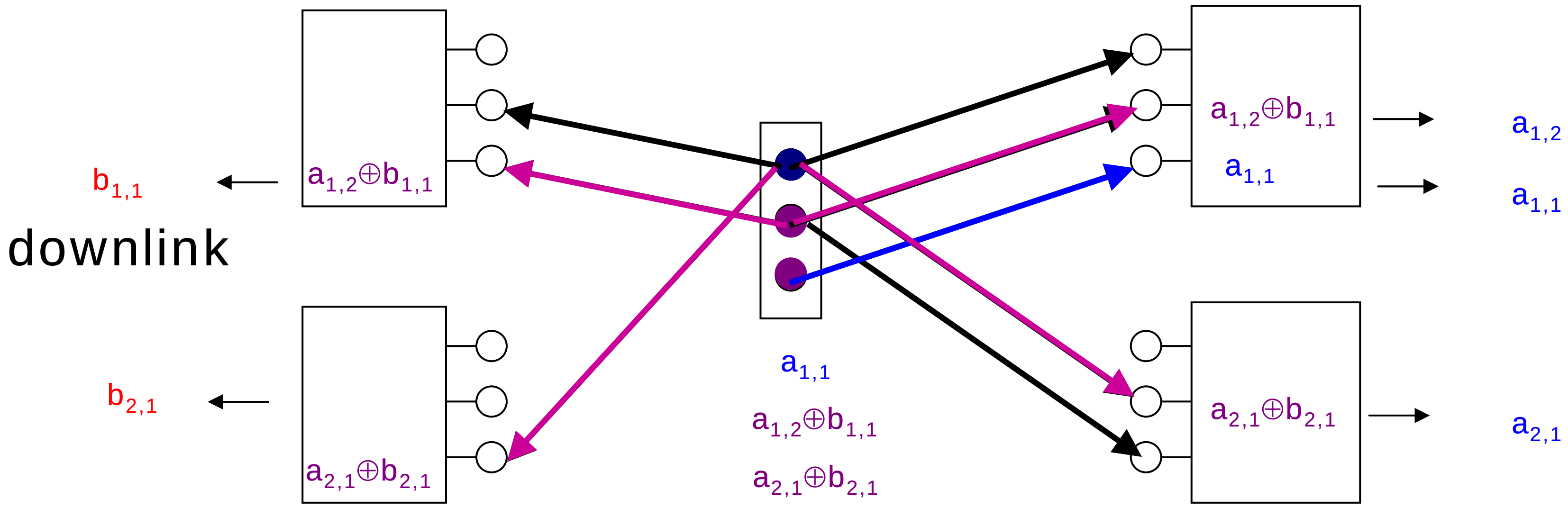
Illustration

- Rate tuple (RA1, RB1,RA2,RB2)=(2,1,1,1) inside the cut-set-region

uplink



Illustration

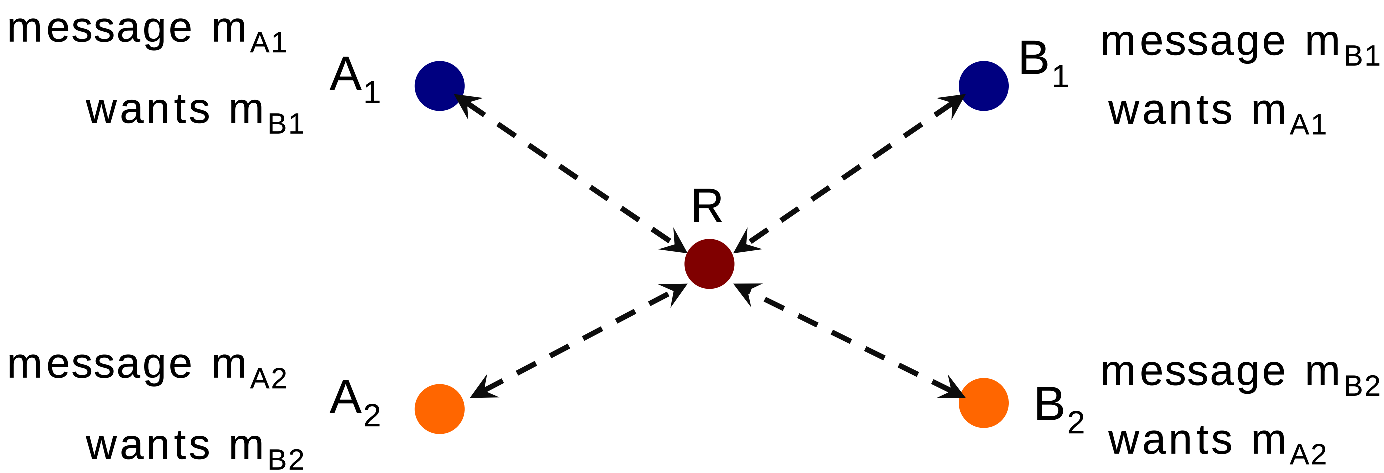


- Achieves rate point (2,1,1,1)
- No interference between different pairs on the same signal level at the relay
- Relay just re-orders and forwards signals

- Gaussian two-pair two-way relay network

Approach

- Does this simple divide-and-conquer work for the relay network?
  - Advantages:
    - Reduced decoding complexity at both the relay and the nodes
    - Limited number of cases to analyze



Transition to Gaussian case

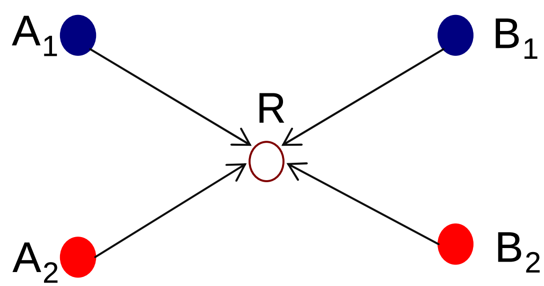
- Three challenges
  - Additive noise (primitive of the Gaussian channel)
  - With superposition: Power leakage from the signals of lower levels to those transmitted at higher levels
  - Decoding the superposition of signals
- Solutions
  - Use appropriate coding scheme
  - Leakage inevitable, however:
    - Compensation in the capacity region for a leakage tolerance (Idea of gap approach)
  - Use appropriate lattice code (superposition of two codewords is a valid codeword)

Relaying scheme: uplink

- Weak users: use a lattice code
- Strong users: use a superposition of a lattice code and a Gaussian code
- **Signal Scale Alignment** at relay using appropriate weights

$$x_{A1} = \sqrt{\alpha_{A1}^{(1)}}\, x_{A1}^{(1)} + \sqrt{\alpha_{A1}^{(2)}}\, x_{A1}^{(2)}$$

$$x_{A2} = \sqrt{\alpha_{A2}^{(1)}}\, x_{A2}^{(1)} + \sqrt{\alpha_{A2}^{(2)}}\, x_{A2}^{(2)}$$



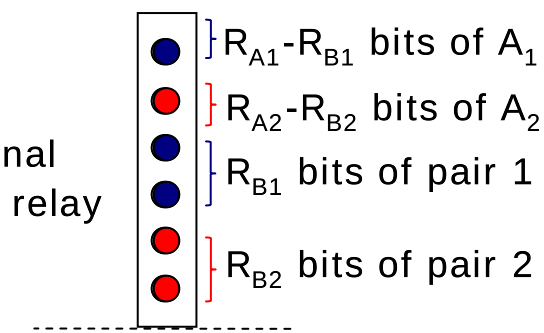
$$x_{B1} = \sqrt{\alpha_{B1}}\, x_{B1}$$

$$x_{B2} = \sqrt{\alpha_{B2}}\, x_{B2}$$

$$|h|_{A1R} \geq |h|_{A2R} \geq |h|_{B1R} \geq |h|_{B2R}$$

Deterministic:

Received signal levels at the relay



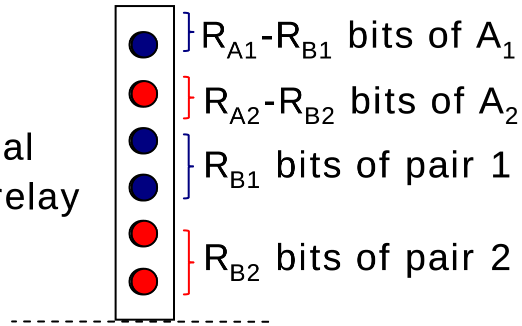
Decoding at the relay

- Relay successively decodes the received signals by treating interference at lower "levels" as noise

1. Decode $x_{A1}^{(1)}$: $R_{A1}^{(1)} \le \log(1+\mathrm{SINR}_{\mathrm{A1}^{(1)}})$
2. Decode $x_{A2}^{(1)}$: $R_{A2}^{(1)} \le \log(1+\mathrm{SINR}_{\mathrm{A2}^{(1)}})$
3. Decode $x_{A1}^{(2)} + x_{B1}$: $R_{A1}^{(2)} = R_{B1} \le \log(\mathrm{SINR}_{\mathrm{A1}^{(2)}})^{+}$
4. Decode $x_{A2}^{(2)} + x_{B2}$: $R_{A2}^{(2)} = R_{B2} \le \log(\mathrm{SINR}_{\mathrm{A2}^{(2)}})^{+}$

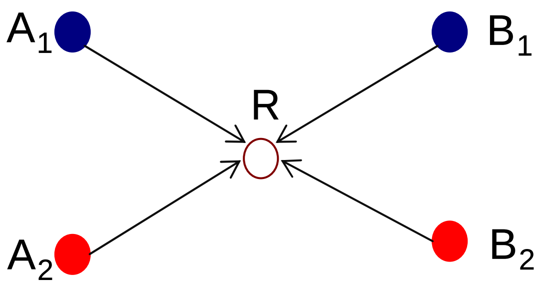
Deterministic:

Received signal levels at the relay

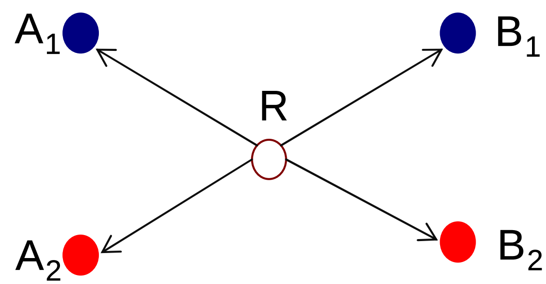


Relaying scheme: downlink

- Relay uses a superposition of four Gaussian codewords to broadcasts the decoded information

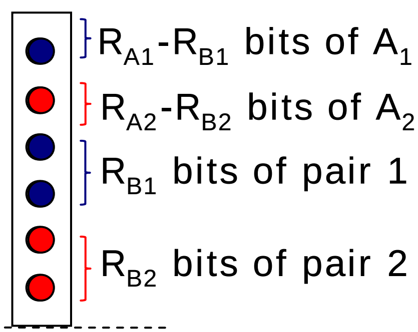


$$|h|_{A1R} \geq |h|_{A2R} \geq |h|_{B1R} \geq |h|_{B2R}$$

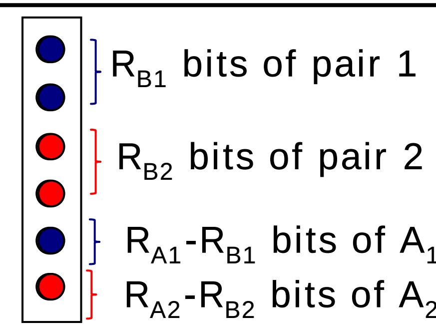


$$|h|_{RB2} \geq |h|_{RB1} \geq |h|_{RA2} \geq |h|_{RA1}$$

Received signal levels at the relay



Transmit signal levels at the relay



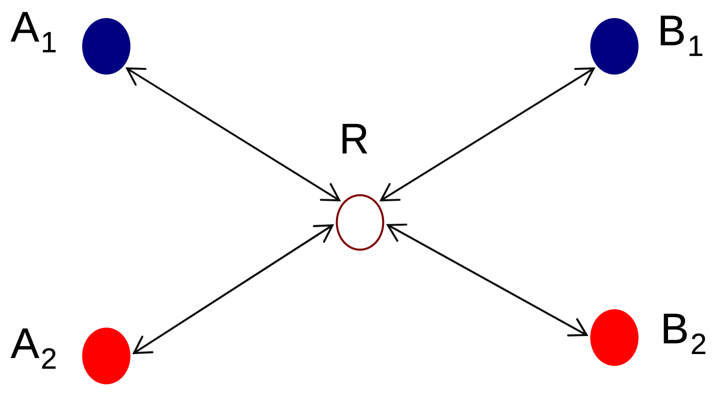
Separation of DL and UL

- Upper bounds given by cut-set bounds
- Uplink and downlink can be analyzed separately
- Without loss of generality
  - 3 cases in uplink, 3 in downlink
  - Assume that R_{A1} is the highest rate
  - Assume that $R_{Ai} \geq R_{Bi} \quad \forall i$

Main Result

- **Theorem:**

  If $R=(R_{A1}, R_{B1}, R_{A2}, R_{B2})$ is in the cut-set region of the two-pair two-way relay network, then $(R_{A1}-2, R_{B1}-2, R_{A2}-2, R_{B2}-2)$ is achievable.



- Optimal within 2 bps/Hz per user

Summary

- Analysis of the capacity region of the bidirectional multi-pair relay network
- Relay processing
  - Decode the sum of lattice codewords and forward
- The divide and conquer with signal level alignment is optimal in the deterministic model
- Gaussian Channel: Characterization of the capacity to within 2 bits/sec/Hz per user

Have I mentioned….

Call for Participation

2010 Workshop on Interference in Networks

March 26, 2010 — Ulm, Germany

The workshop will take place on March 26, 2010 in Ulm. Ulm is located in the southern part of Germany between Munich and Stuttgart. Traveling by train takes around 2 hours from Munich and Frankfurt and less from Stuttgart. The workshop is planned to start at 9am and will end by 5pm.

Interference is one of the defining impairments in wireless networks. Despite the huge efforts in research even the most simple interference setups are not completely characterized in terms of capacity and other fundamental limits. Recently, however, some new and promising techniques such as interference alignment and deterministic models have been proposed resulting in a burst of research activities on interference networks. The idea of the workshop is thus to bring researchers together to share ideas and results on the following topics (but not limited to)

- Generalized degrees of freedom
- Structured codes
- Approximate capacity characterizations
- Cross-layer design in interference networks
- Distributed algorithms for interference management
- Cooperative, cognitive and competitive strategies
- PHY layer security in networks
- Signal Processing in Networks

Upper bound

$$\overline{\mathcal{C}} = \Big\{ (R_{A_1}, R_{B_1}, R_{A_2}, R_{B_2}) \in \mathbb{R}_+^4 :$$

$$R_{A_i} \leq \min \left(C\left(|h_{A_iR}|^2\right), C\left(|h_{RB_i}|^2\right)\right) \tag{1}$$

$$R_{B_i} \leq \min \left(C\left(|h_{B_iR}|^2\right), C\left(|h_{RA_i}|^2\right)\right) \tag{2}$$

$$R_{A_1} + R_{A_2} \leq \min \Big(C\left(|h_{A_1R}|^2 + |h_{A_2R}|^2\right), C\left(\max\left(|h_{RB_1}|^2, |h_{RB_2}|^2\right)\right) \Big) \tag{3}$$

$$R_{B_1} + R_{B_2} \leq \min \Big(C\left(|h_{B_1R}|^2 + |h_{B_2R}|^2\right), C\left(\max\left(|h_{RA_1}|^2, |h_{RA_2}|^2\right)\right) \Big) \tag{4}$$

$$R_{A_1} + R_{B_2} \leq \min \Big(C\left(|h_{A_1R}| + |h_{B_2R}|^2\right), C\left(\max\left(|h_{RB_1}|^2, |h_{RA_2}|^2\right)\right) \Big) \tag{5}$$

$$R_{B_1} + R_{A_2} \leq \min \Big(C\left(|h_{B_1R}|^2 + |h_{A_2R}|^2\right), C\left(\max\left(|h_{RA_1}|^2, |h_{RB_2}|^2\right)\right) \Big) \Big\}, \tag{6}$$

where $C(x) = \log(1+x)$.

Uplink-region $\mathcal{C}_u$: The set of rates satisfying (1)-(6) with down-link gains assumed infinity

Likewise for $\mathcal{C}_d$, the downlink-region

$$\text{Then: } \bar{\mathcal{C}} = \mathcal{C}_d \cap \mathcal{C}_u$$